Neutrino Mass Bound from Cosmological Probes

(LCDM vs Interacting Dark-Energy Model)

Yong-Yeon Keum
Seoul National University

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- Neutrino Masses from Large Scale Structures (CMB, Power Spectrum,.....)

  Lambda CDM vs INuDE-Model
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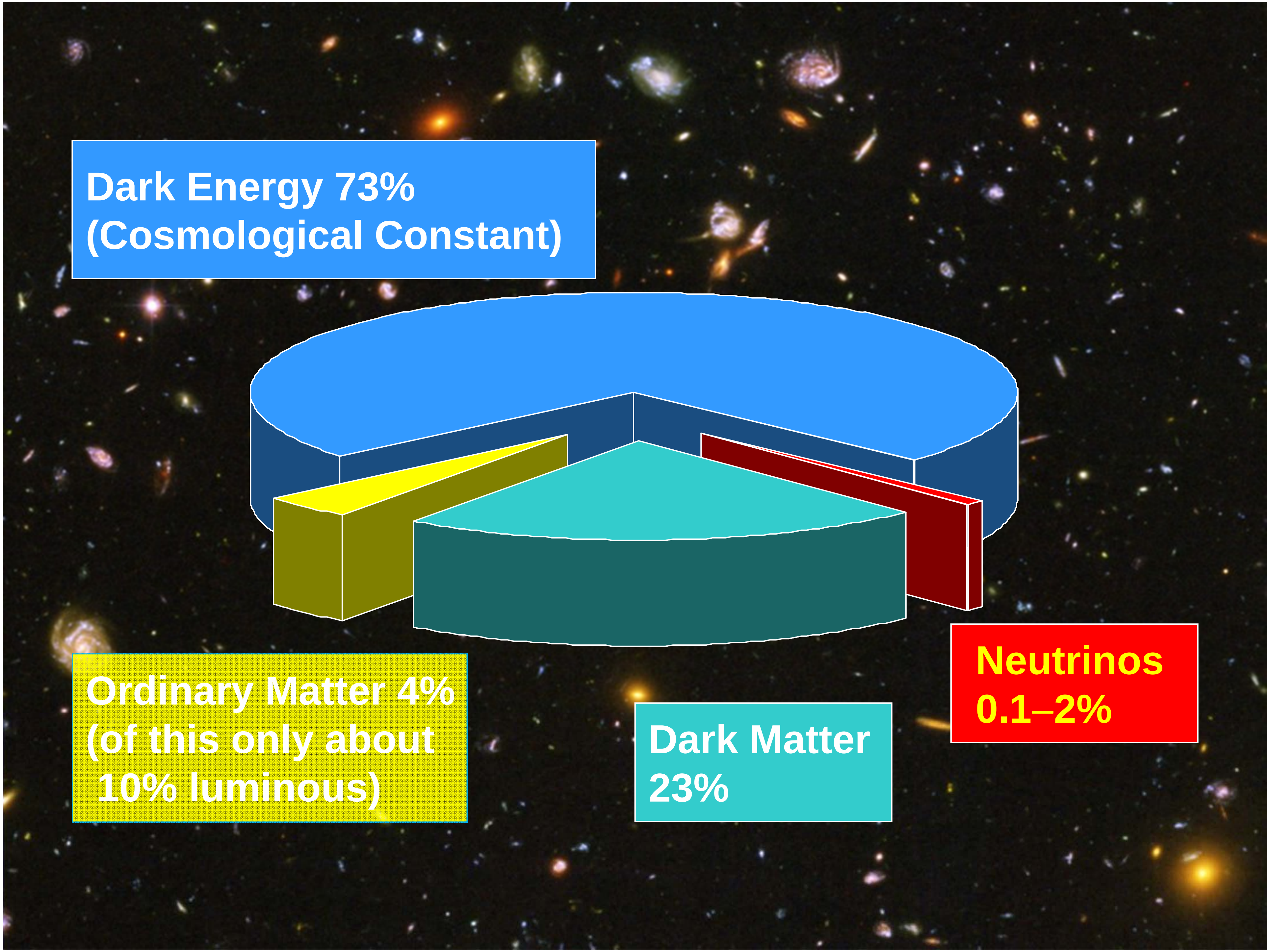
Papers: YYK and K. Ichiki, JCAP 0806, 005, 2008;
JHEP 0806, 058, 2008;
arXiv:0803.3142, and in preparing
for WMAP-7 year data

References:

Massive Neutrinos and Cosmology: J. Lesgourgues and S. Pastor, Phys. Rep. 429:307(2006)


Dark Energy 73%
(Cosmological Constant)
Ordinary Matter 4%
(of this only about
10% luminous)
Dark Matter
23%
Neutrinos
0.1–2%

What we know right Now:

- neutrinos have mass (NuOsc-exp.)
- the rough magnitude of the leptonic mixing angles (two large and one relatively small angles)
- the masses of all three neutrino species are very small compared with charged fermions

What we don't know:

- Are neutrinos their own anti-particles ?
  (Dirac vs Majorana particles)

- What is the absolute mass of neutrinos and their mass ordering, i.e.
  (normal, inverted or quasi-degenerate ?)

- Is there CP violation in the leptonic sector ?

- If the 0νββ decay will be observed and

$$0.42\sqrt{\Delta m^2_{atm}} \;\leq\; |m_{\beta\beta}| \;\leq\; \sqrt{\Delta m^2_{atm}}$$

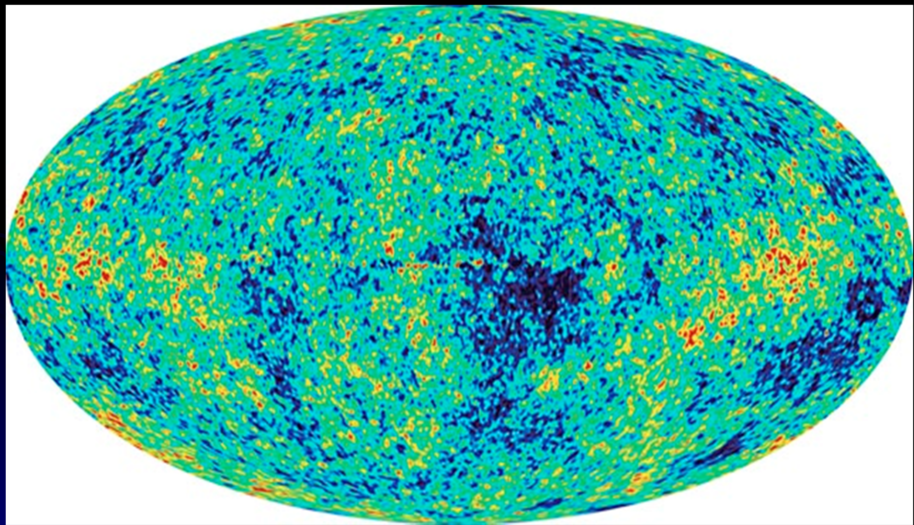
it will be an indication of the inverted hierarchy

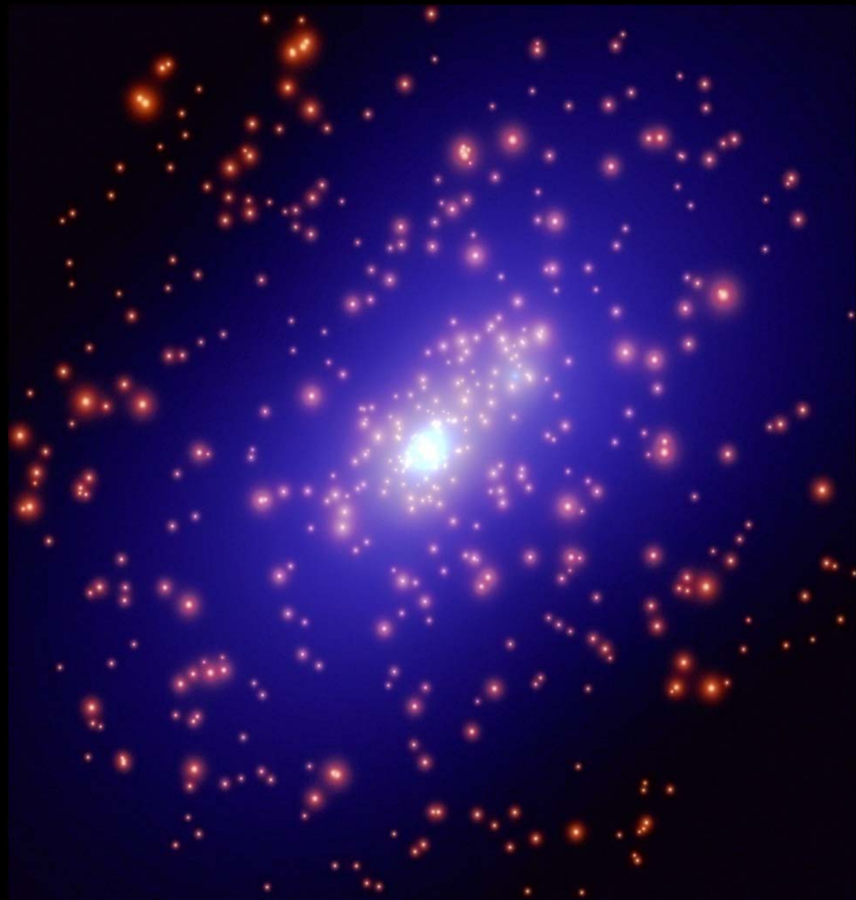
- Normal Hierarchy : M_nu > 0.03 eV
- Inverted Hierarchy: M_nu > 0.07 eV

Remarks: It is really difficult to confirm the normal hierarchy in neutrinoless double beta decay in future experiments.

How can we reach there ?

Neutrino Mass bound from Large Scale Structures (CMB, Power Spectrum,…..)





Neutrino free-stream :

- If ρ_ν is carried by free-moving relativistic particles, we can discriminate between massless vs massive ,and between free vs interacting neutrinos.

- Neutrino masses determine two-different things:

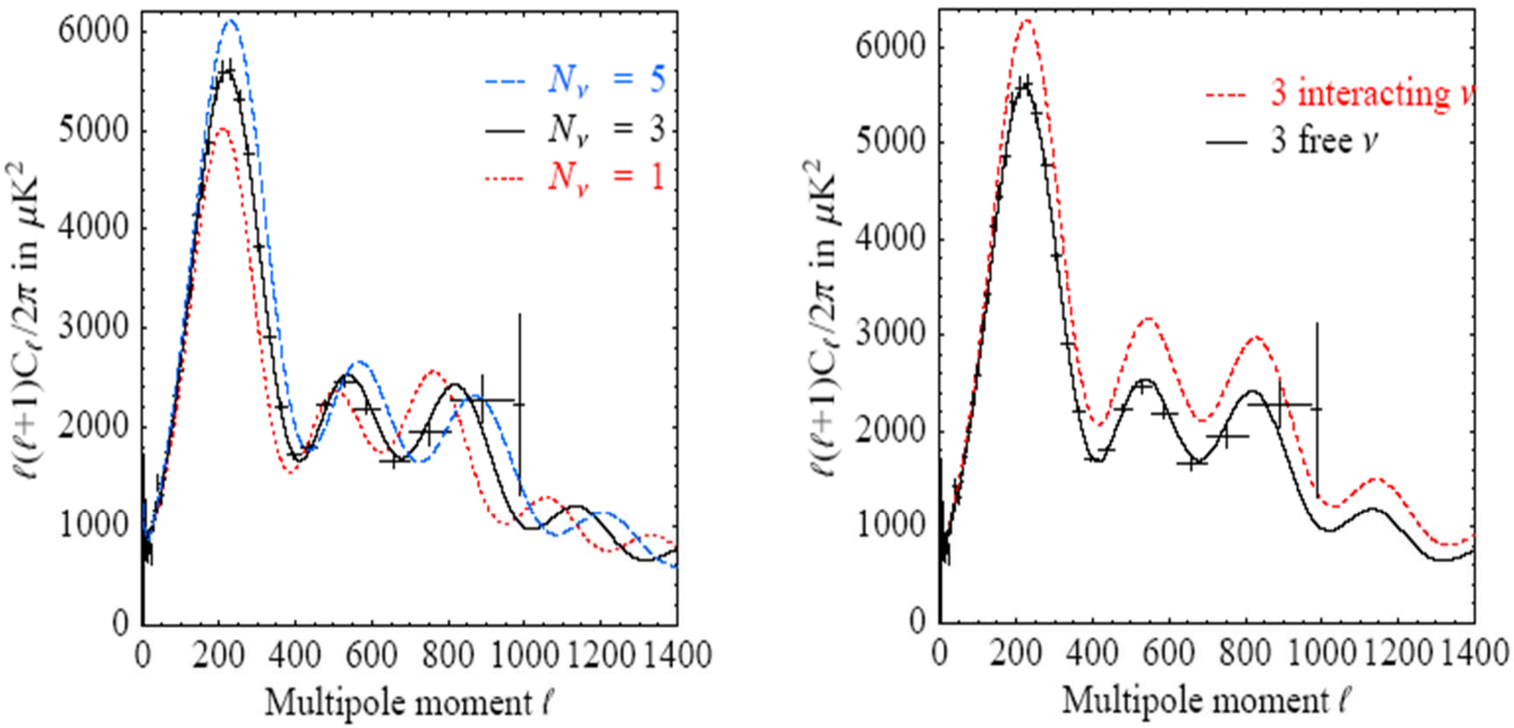
1) temperature at which neutrinos cease to be non-relativistic, which controls the length on which neutrinos travel reducing clustering.

2) the function of energy carried by neutrinos, which controls how much neutrinos can smooth inhomogeneities.

- In standard cosmology:

$$\Omega_\nu h^2 \lesssim 0.6 \cdot 10^{-2} \qquad \text{i.e.} \qquad \sum m_{\nu_i} \lesssim 0.6\,\text{eV} \qquad \text{at } 99.9\% \text{ C.L.}$$

CMB vs Nν



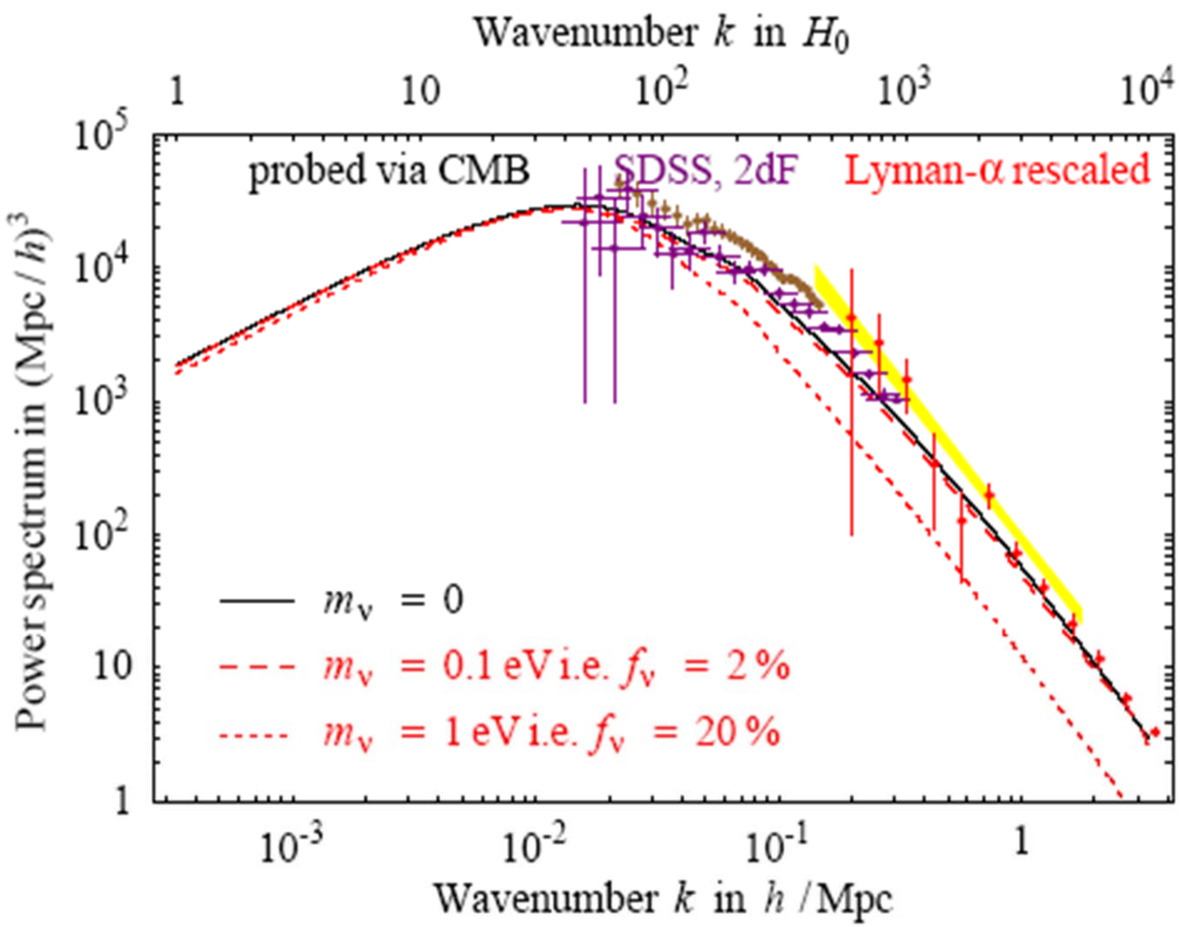
6000
5000
4000
3000
2000
1000
0
$\ell(\ell+1)C_\ell/2\pi$ in μK^2
N_ν = 5
N_ν = 3
N_ν = 1
0 200 400 600 800 1000 1200 1400
Multipole moment ℓ
3 interacting ν
3 free ν

Neutrino mass effects

- After neutrinos decoupled from the thermal bath, they stream freely and their density pert. are damped on scale smaller than their free streaming scale.
- The free streaming effect suppresses the power spectrum on scales smaller than the horizon when the neutrino become non-relativistic.
- Δ Pm(k)/Pm(k) = -8 Ω_ν / Ω_m

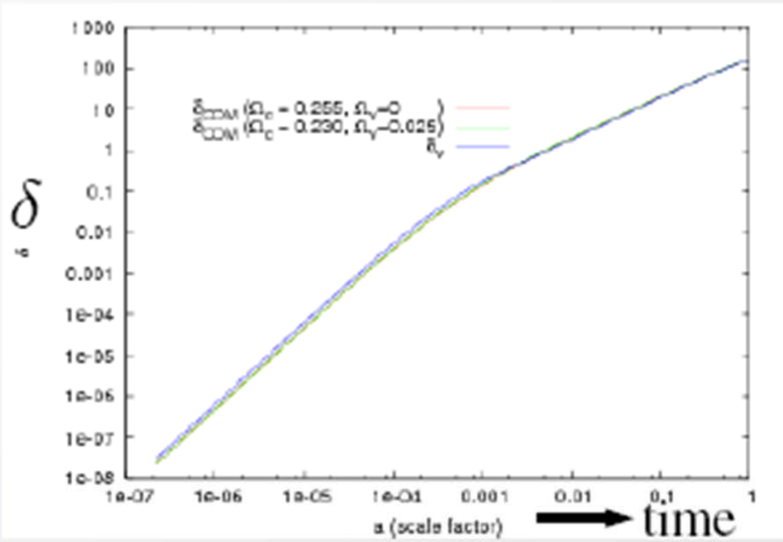
- Analysis of CMB data are not sensitive to neutrino masses if neutrinos behave as massless particles at the epoch of last scattering. Neutrinos become non-relativistic before last scattering when Ω_ν h^2 > 0.017 (total nu. Masses > 1.6 eV). Therefore the dependence of the position of the first peak and the height of the first peak has a turning point at Ω_ν h^2 = 0.017.

Mass Power spectrum vs Neutrino Masses

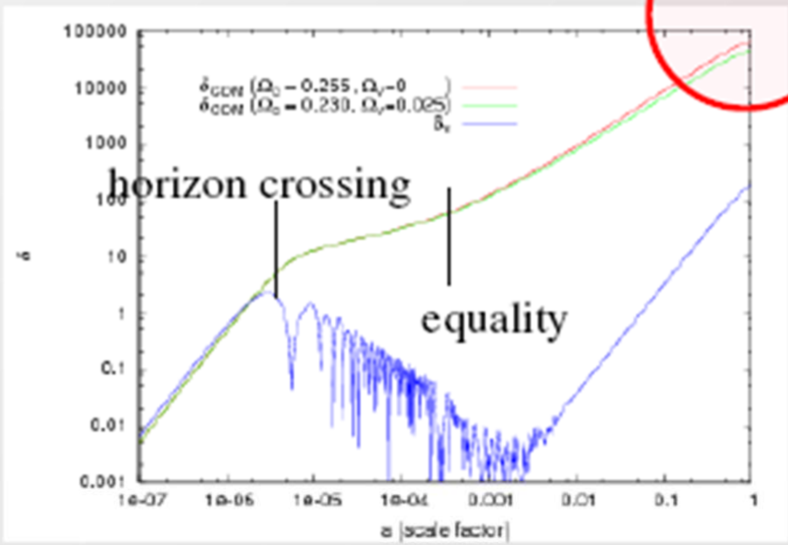


Evolution of density perturbations in the presence of massive neutrinos

$k = 3.0 \times 10^{-3}\mathrm{Mpc}^{-1}$



$k = 3.0\mathrm{Mpc}^{-1}$



$$\delta(x) = \frac{\delta\rho(x)}{\rho} = \int d^3k\,\delta(k)e^{ik\cdot x}$$ Fourier mode of density fluctuation

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\left(\rho_{\mathrm{cdm}}\delta_{\mathrm{cdm}} + \rho_\nu\delta_\nu\right)$$ at large scales

$$\ddot{\delta} + \underbrace{2H\dot{\delta}}_{\text{friction due to expansion}} = 4\pi G\left(\underbrace{\rho_{\mathrm{cdm}}\delta_{\mathrm{cdm}}}_{\text{gravitational force}} \cancel{+ \rho_\nu\delta_\nu}\right)$$ at small scales

Power spectrum

$P_m(k,z) = P_*(k) \quad T^2(k,z)$ — Transfer Function:

$T(z,k) := \delta(k,z)/[\delta(k,z=z_*)D(z_*)]$

Primordial matter power spectrum (Ak^n)

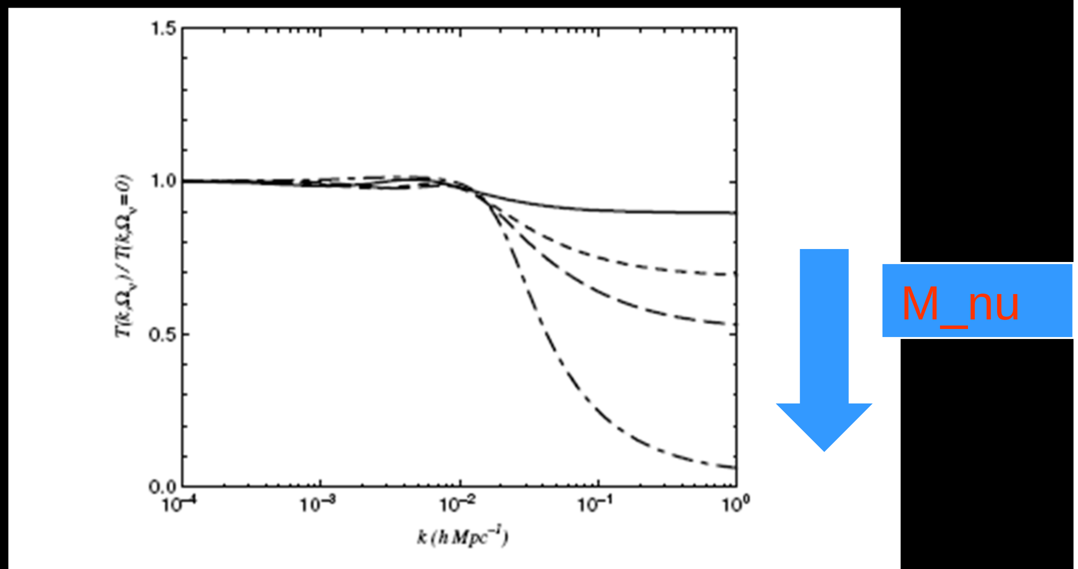
z_*:= a time long before the scale of interested have entered in the horizon

Large scale: $T \sim 1$

Small scale : $T \sim 0.1$

$\Delta P_m(k)/P_m(k) \sim -8\ \Omega_\nu/\Omega_m$

$= -8\ f_\nu$



Numerical Analysis

MCMC likelihood analysis

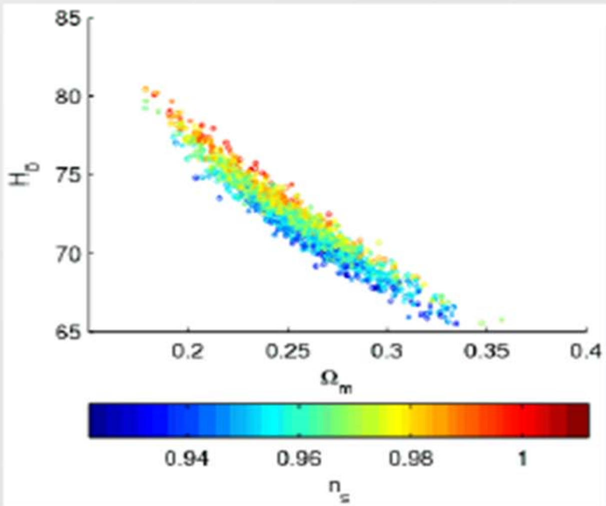
- cosmological parameters (7 params)

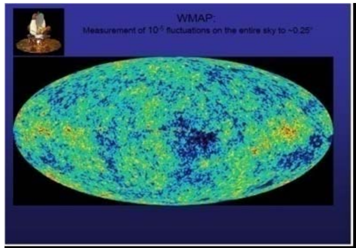
$$\vec{P} = (\Omega_b h^2, \Omega_c h^2, \theta, \tau, m_\nu, n_s, A_s)$$

- explore the likelihoods of WMAP5 and CFHTLS data using Markov Chain Monte Calro sampling

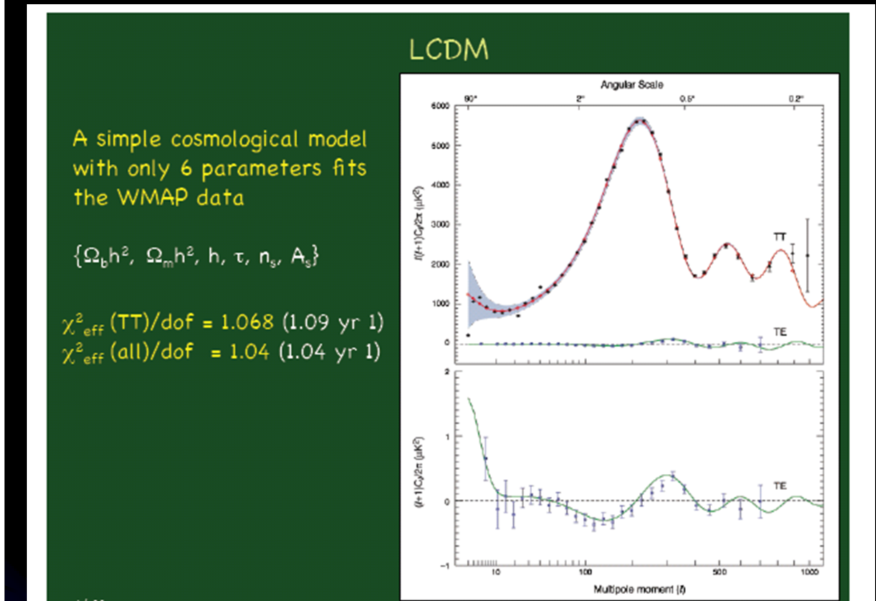
- CosmoMC: Cosmological MCMC engine

(http://cosmologist.info/cosmomc)

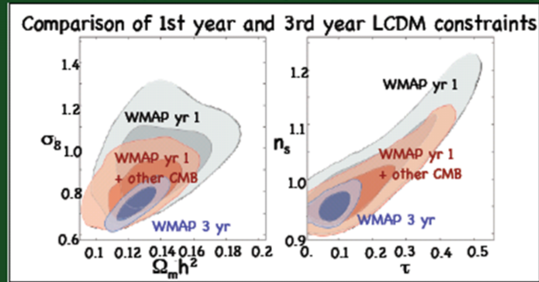




Experimental Obs.(WMAP)



Improvement in Parameters from EE tau measurement



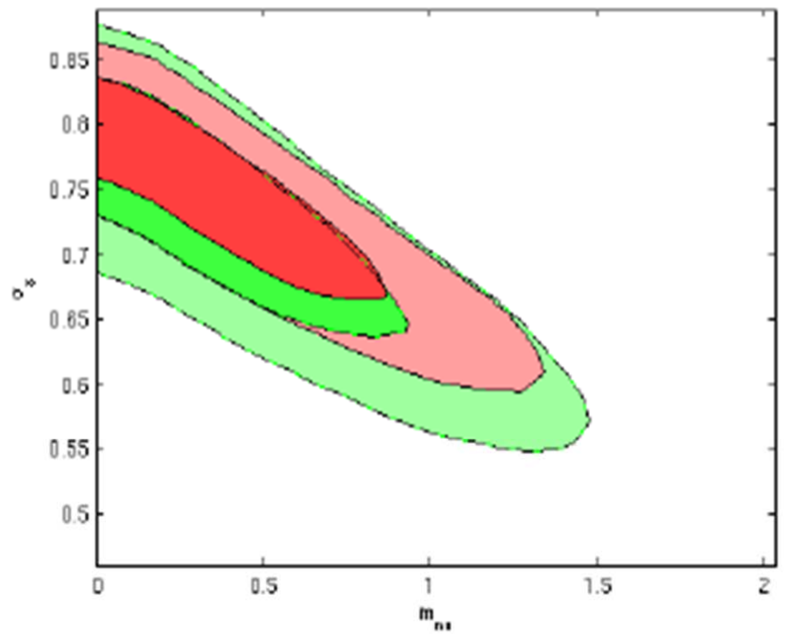
Parameter	First Year Mean	WMAPext Mean	Three Year Mean	First Year ML	WMAPext ML	Three Year ML
$100\Omega_b h^2$	$2.38^{+0.13}_{-0.12}$	$2.32^{+0.12}_{-0.11}$	2.23 ± 0.08	2.30	2.21	2.22
$\Omega_m h^2$	$0.144^{+0.016}_{-0.016}$	$0.134^{+0.006}_{-0.006}$	0.126 ± 0.009	0.145	0.138	0.128
H_0	72^{+5}_{-5}	73^{+3}_{-3}	74^{+3}_{-3}	68	71	73
τ	$0.17^{+0.08}_{-0.07}$	$0.15^{+0.07}_{-0.07}$	0.093 ± 0.029	0.10	0.10	0.092
n_s	$0.99^{+0.04}_{-0.04}$	$0.98^{+0.03}_{-0.03}$	0.961 ± 0.017	0.97	0.96	0.958
Ω_m	$0.29^{+0.07}_{-0.07}$	$0.25^{+0.03}_{-0.03}$	0.234 ± 0.035	0.32	0.27	0.24
σ_8	$0.92^{+0.1}_{-0.1}$	$0.84^{+0.06}_{-0.06}$	0.76 ± 0.05	0.88	0.82	0.77

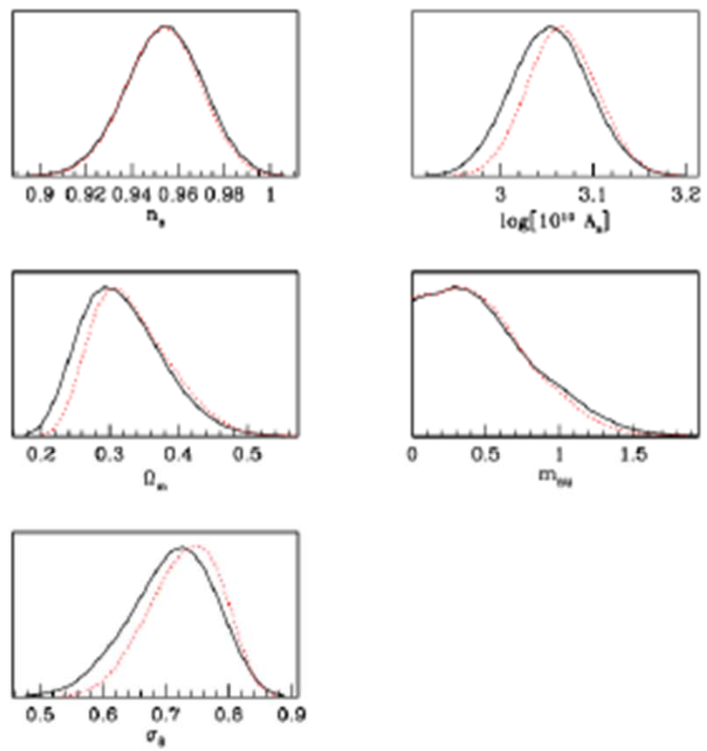
n_s: Spectral index
tau: optical depth
sigma_8: rms fluctuation parameter
A_s: the amp. of the primordial scalar power spectrum

$$\sigma_8 = \langle \left(\frac{\delta M}{M}\right)^2 \rangle = \int dk\, 4\pi k^2 P(k) [3 \frac{\sin(kr) - kr\cos(kr)}{(kr)^2}]^2$$

Comparison of the results

- Green (WMAP5), Red(+CFHTLS)





WMAP5 -- $m_\nu < 1.2$ eV

WMAP5+CFHTLS -- $m_\nu < 1.1$ eV

Within Standard Cosmology Model (LCDM)

Upper limits on neutrino masses from Cosmology

Assume that the underlying cosmological model is:

- the standard spatially flat ΛCDM model with adiabatic primodial perturbations,
- they have no non-standard interactions,
- they decouple from the thermal background at temperatures of order 1 MeV,
- use the relation between the sum of the neutrino masses and their contribution to the energy density of the universe is:

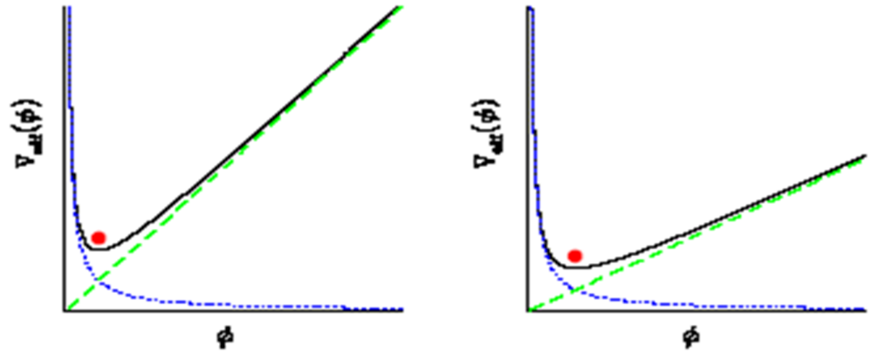
$$\Omega_\nu \; h^2 = M_\nu / 93.14 \; eV \tag{1}$$

Data	Authors	M_ν-bound
2dFGRS (P01)	Elgarøy et al. [2002]	1.8 eV
CMB+2dFGRS(C05)	Sanchez et al. [2005]	1.2 eV
CMB+LSS+SNIa+BAO	Goobar et al. [2006]	0.62 eV
WMAP (3 year) alone	Fukugita et al. [2006]	2.0 eV
CMB+LSS+SNIa	Spergel et al. [2006]	0.68 eV
CMB + LSS+SNIa+BAO+Lyα	Seljak et al. [2006]	0.17 eV

Table 2: Some recent cosmological neutrino mass bounds (95 % CL).

What is the upper bound of neutrino masses beyond Lambda CDM Model ?

Example: Interacting Neutrino-Dark-Energy Model



At low energy,

$$\mathcal{L} = m_\nu \bar{\nu}^c \nu + V_{tot}(m_\nu)\,,$$

where $V_{tot}(m_\nu) = V_\nu(m_\nu) + V_0(m_\nu)$

$n_\nu\, m_\nu(\phi)$

Scalar potential in vacuum

The condition of minimization of V_{tot} determines the physical neutrino mass.

Background Equations:

K. Ichiki and YYK:2007

Equations for quintessence scalar field are given by

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 \frac{dV_{\rm eff}(\phi)}{d\phi} = 0 , \qquad (1)$$

$$V_{\rm eff}(\phi) = V(\phi) + V_{\rm I}(\phi) , \qquad (2)$$

$$V_{\rm I}(\phi) = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2(\phi)} f(q) , \qquad (3)$$

$$m_\nu(\phi) = \bar{m}_i e^{\beta \frac{\phi}{M_{\rm pl}}} \text{ (as an example)}, \qquad (4)$$

Energy densities of mass varying neutrino (MVN) and quintessence scalar field are described as

$$\rho_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2} f_0(q) , \qquad (5)$$

$$3P_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \frac{q^2}{\sqrt{q^2 + a^2 m_\nu^2}} f_0(q) , \qquad (6)$$

$$\rho_\phi = \frac{1}{2a^2} \dot{\phi}^2 + V(\phi) , \qquad (7)$$

$$P_\phi = \frac{1}{2a^2} \dot{\phi}^2 - V(\phi) . \qquad (8)$$

From equations (5) and (6), the equation of motion for the background energy density of neutrinos is given by

$$\dot{\rho}_\nu + 3\mathcal{H}(\rho_\nu + P_\nu) = \frac{\partial \ln m_\nu}{\partial \phi} \dot{\phi} (\rho_\nu - 3P_\nu) . \qquad (9)$$

Perturbation Equations:

We consider the linear perturbation in the synchronous Gauge and the linear elements:

$$ds^2 = a^2(\tau) \left[-d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right] , \qquad (10)$$

1 Boltzmann Equation for Mass Varying Neutrino

Here we have splitted the comoving momentum q_j into its magnitude and direction: $q_j = qn_j$, where $n^i n_i = 1$.
The Boltzmann equation is

$$\frac{Df}{D\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = \left(\frac{\partial f}{\partial \tau}\right)_C . \quad (34)$$

in terms of these variables. From the time component of geodesic equation,

$$\frac{1}{2}\frac{d}{d\tau}\left(P^0\right)^2 = -\Gamma^0_{\alpha\beta}P^\alpha P^\beta - mg^{0\nu}m_{,\nu} , \quad (35)$$

and the relation $P^0 = a^{-2}\epsilon = a^{-2}\sqrt{q^2 + a^2 m_\nu^2}$, we have

$$\frac{dq}{d\tau} = -\frac{1}{2}\dot{h}_{ij}qn^i n^j - a^2\frac{m}{q}\frac{\partial m}{\partial x^i}\frac{dx^i}{d\tau} . \quad (36)$$

We will write down each term up to $\mathcal{O}(h)$:

$$\begin{aligned} \frac{\partial f}{\partial \tau} &= \frac{\partial f_0}{\partial \tau} + f_0\frac{\partial \Psi}{\partial \tau} + \frac{\partial f_0}{\partial \tau}\Psi \\ \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} &= \frac{q}{\epsilon}n^i \times f_0\frac{\partial \Psi}{\partial x^i} , \end{aligned}$$

unperturbed Fermi-Dirac distribution by q. Thus we have

$$f_0 = f_0(\epsilon) = \frac{g_s}{h_{\mathrm{P}}^3} \frac{1}{e^{q/k_{\mathrm{B}}T_0} + 1} \ , \tag{40}$$

whish can also be a solution of eq.(38).

1.2 perturbation equations

The first-order Boltzmann equation is

$$\begin{aligned} \frac{\partial \Psi}{\partial \tau} \quad &+ \quad i\frac{q}{\epsilon}(\hat{\boldsymbol{n}} \cdot \boldsymbol{k})\Psi + \left(\dot{\eta} - (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}})^2 \frac{\dot{h} + 6\dot{\eta}}{2}\right) \frac{\partial \ln f_0}{\partial \ln q} \\ &- \quad i\frac{q}{\epsilon}(\hat{\boldsymbol{n}} \cdot \boldsymbol{k})k\delta\phi \frac{a^2 m^2}{q^2} \frac{\partial \ln m}{\partial \phi} \frac{\partial \ln f_0}{\partial \ln q} = 0 \ . \end{aligned} \tag{41}$$

Following previous studies, we shall assume that the initial momentum dependence is axially symmetric so that Ψ depends on $\boldsymbol{q} = q\hat{\boldsymbol{n}}$ only through q and $\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}}$. With this assumption, we expand the perturbation of distribution function, Ψ, in a Legendre series,

$$\Psi(\boldsymbol{k}, \hat{\boldsymbol{n}}, q, \tau) = \sum (-i)^\ell (2\ell + 1) \Psi_\ell(\boldsymbol{k}, q, \tau) P_\ell(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}}) \ . \tag{42}$$

Then we obtain the hierarchy for MVN

$$\dot{\Psi}_0 = -\frac{q}{\epsilon}k\Psi_1 + \frac{\dot{h}}{6}\frac{\partial \ln f_0}{\partial \ln q} , \tag{43}$$

$$\dot{\Psi}_1 = \frac{1}{3}\frac{q}{\epsilon}k\left(\Psi_0 - 2\Psi_2\right) + \kappa , \tag{44}$$

$$\dot{\Psi}_2 = \frac{1}{5}\frac{q}{\epsilon}k(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{\partial \ln f_0}{\partial \ln q} , \tag{45}$$

$$\dot{\Psi}_\ell = \frac{q}{\epsilon}k\left(\frac{\ell}{2\ell+1}\Psi_{\ell-1} - \frac{\ell+1}{2\ell+1}\Psi_{\ell+1}\right) . \tag{46}$$

where

$$\kappa = -\frac{1}{3}\frac{q}{\epsilon}k\frac{a^2m^2}{q^2}\delta\phi\frac{\partial \ln m_\nu}{\partial \phi}\frac{\partial \ln f_0}{\partial \ln q} . \tag{47}$$

Here we used the recursion relation

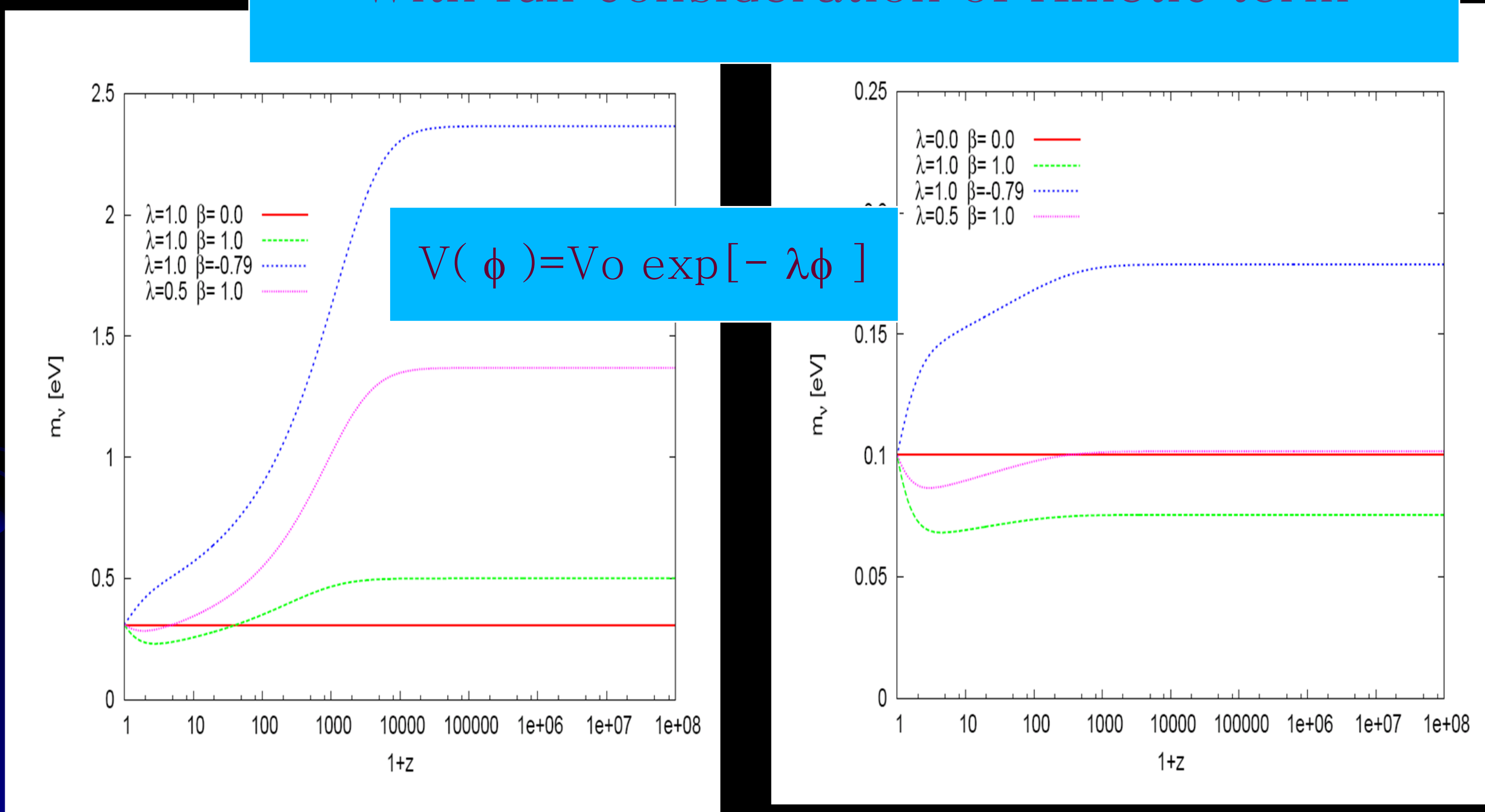
$$(\ell+1)P_{\ell+1}(\mu) = (2\ell+1)\mu P_\ell(\mu) - \ell P_{\ell-1}(\mu) . \tag{48}$$

We have to solve these equations with a q-grid for every wavenumber k...

Varying Neutrino Mass

With full consideration of Kinetic term

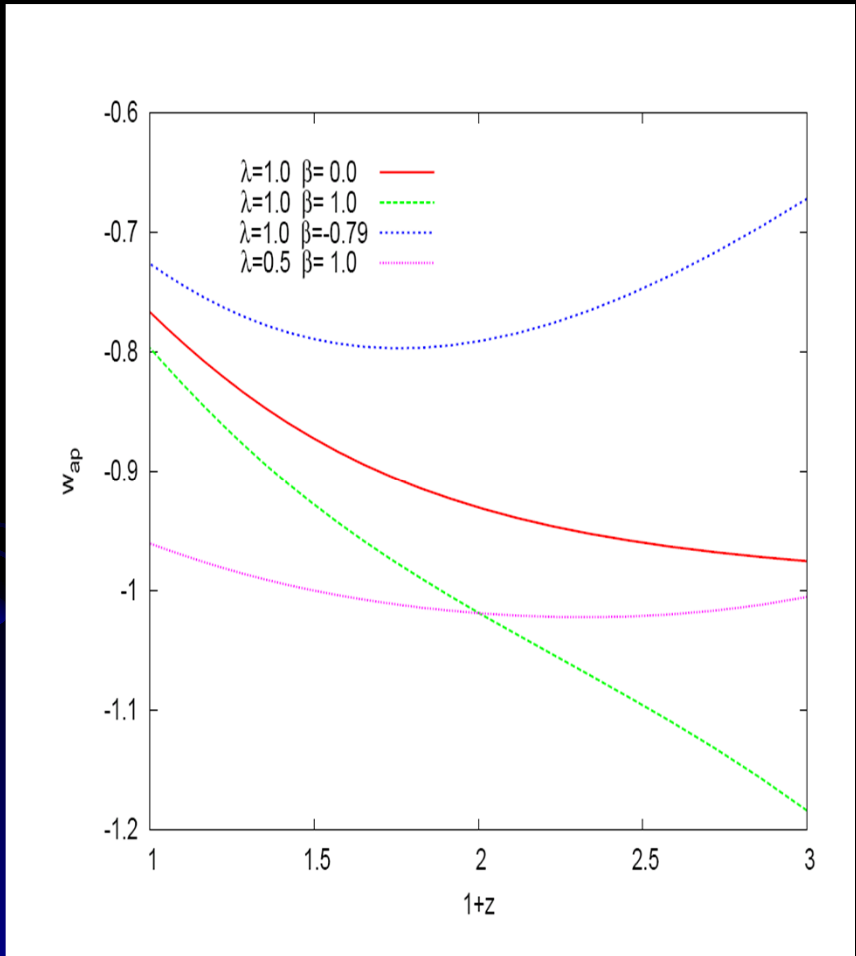
$V(\phi)=V_o \exp[-\lambda\phi]$

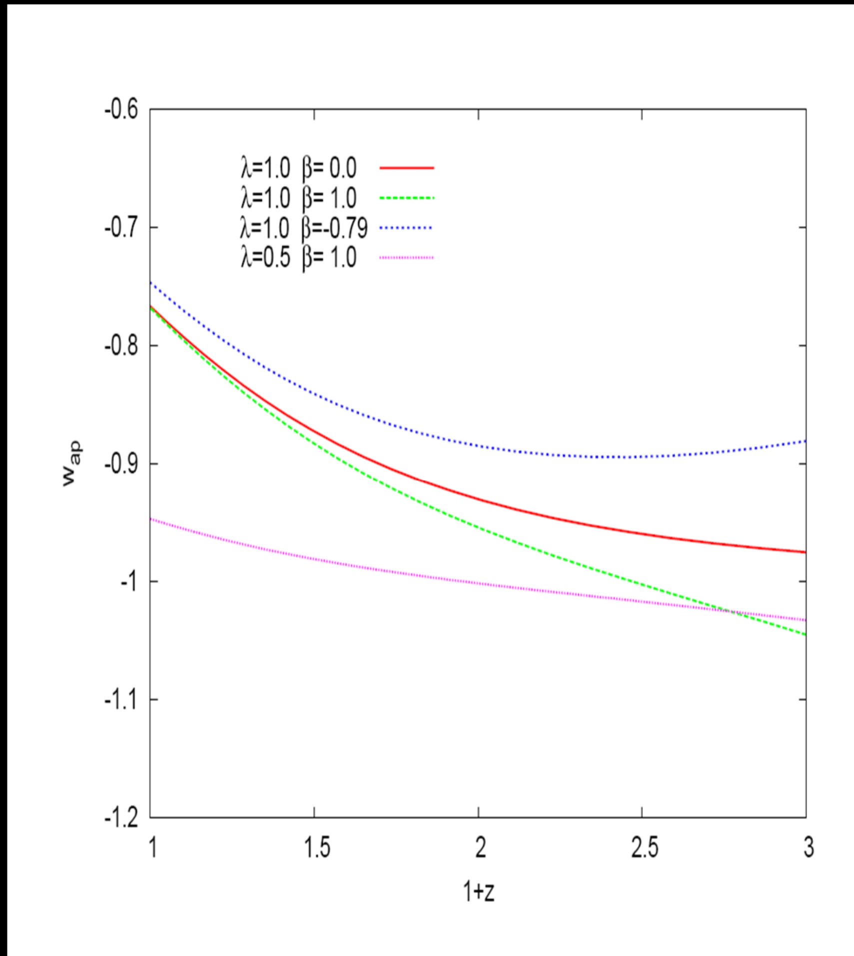


Mν=0.9 eV

Mν=0.3 eV

W_eff





Mν=0.9 eV

Mν=0.3 eV

Neutrino Masses vs z

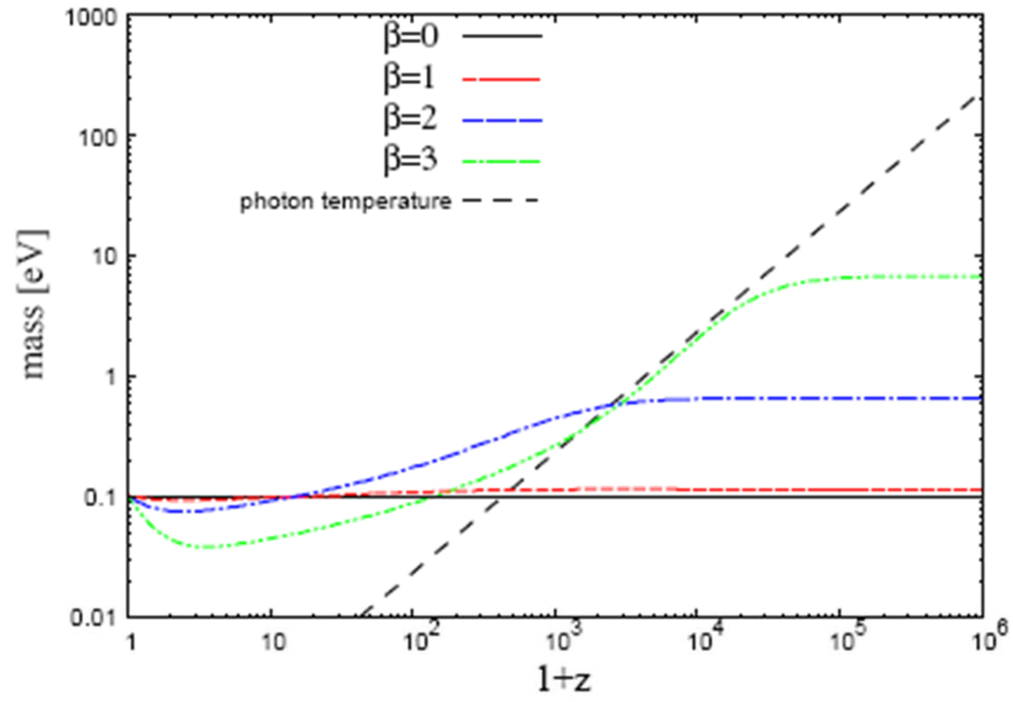
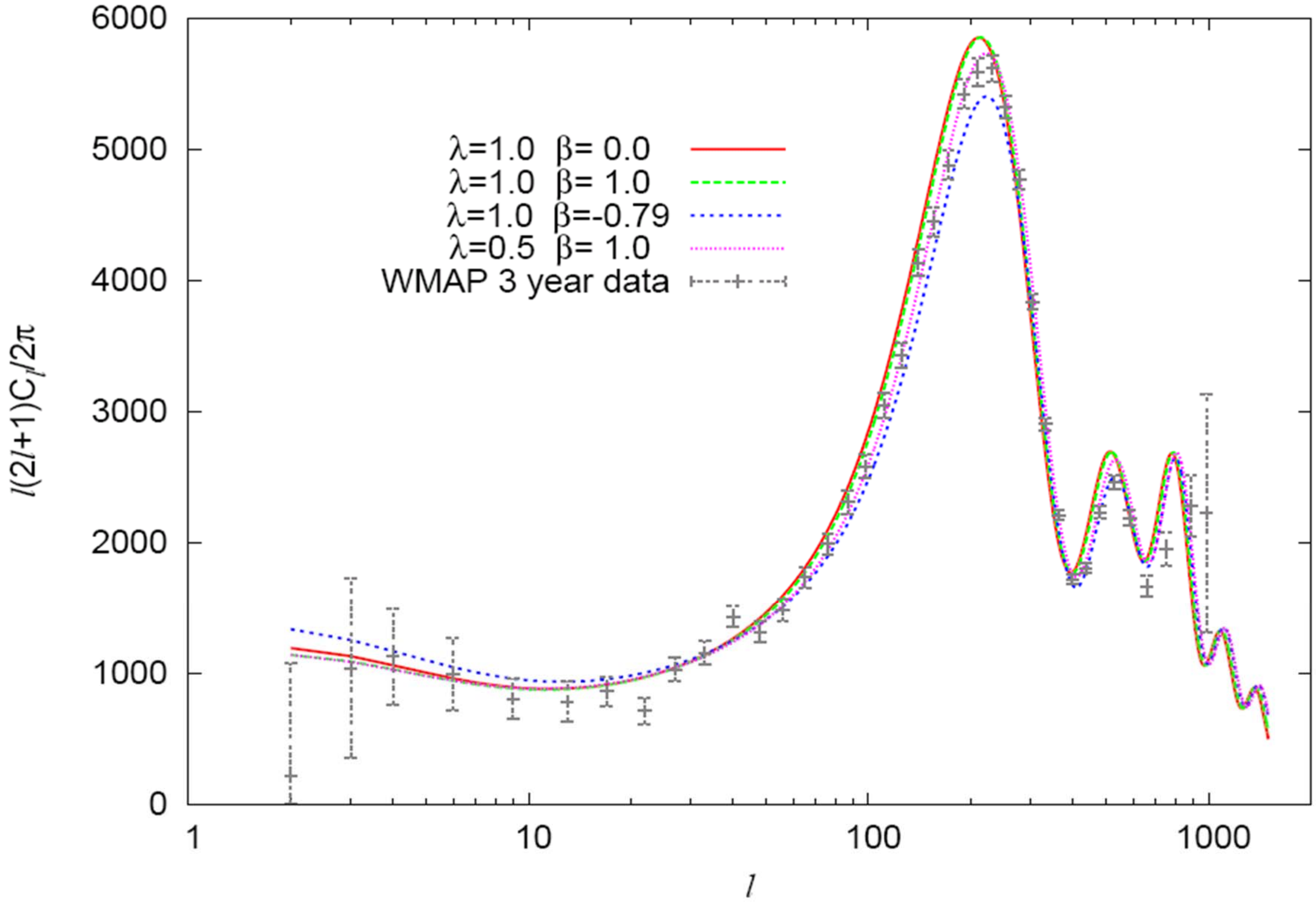
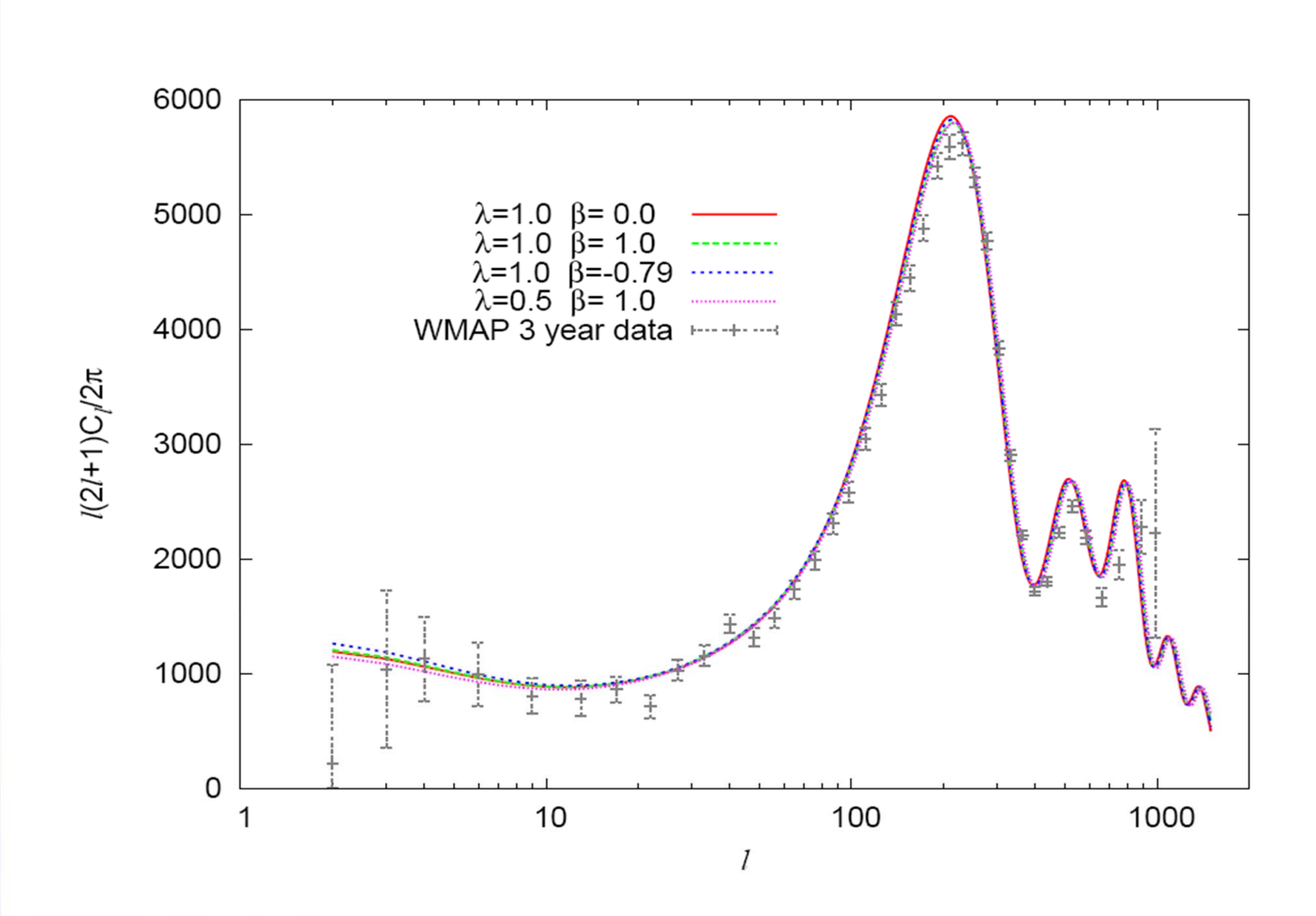


Figure 5: Examples of the time evolution of neutrino mass in power law potential models (Model I) with $\alpha = 1$ and $\beta = 0$ (black solid line), $\beta = 1$ (red dashed line), $\beta = 2$ (blue dash-dotted line), $\beta = 3$ (dash-dot-dotted line). The larger coupling parameter leads to the larger mass in the early universe.



6000
5000
4000
3000
2000
1000
0
1
10
100
1000
$l(2l+1)C_l/2\pi$
l
λ=1.0 β= 0.0
λ=1.0 β= 1.0
λ=1.0 β=-0.79
λ=0.5 β= 1.0
WMAP 3 year data

Mν=0.9 eV



6000
5000
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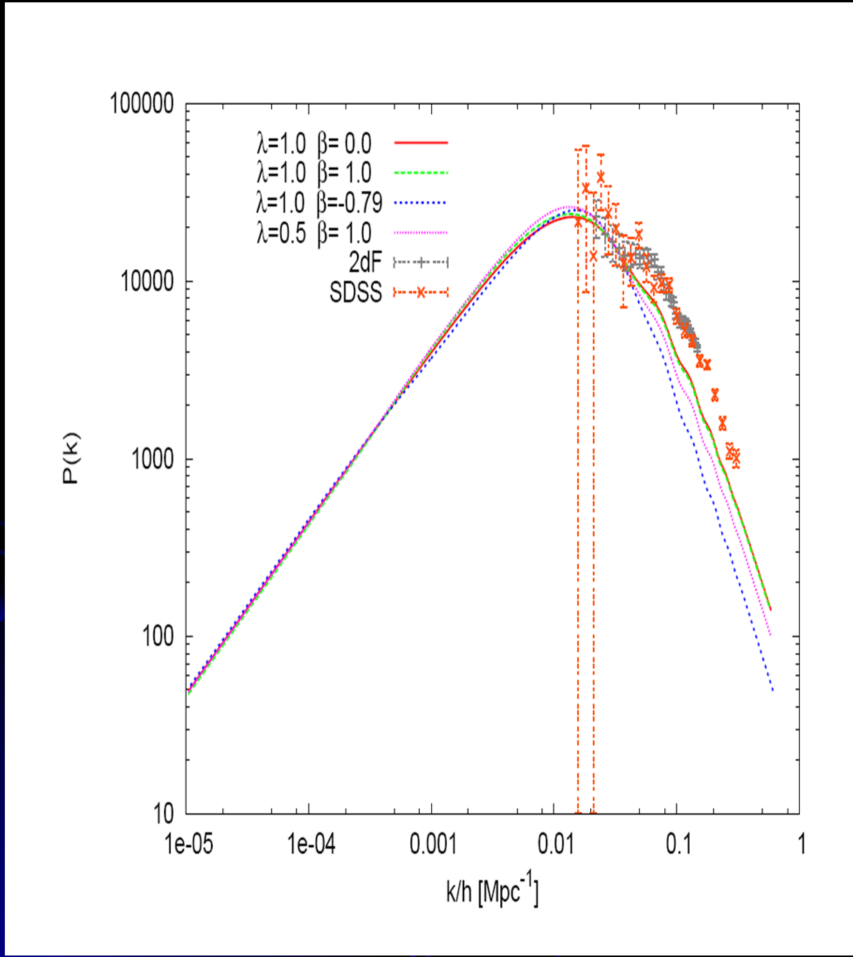
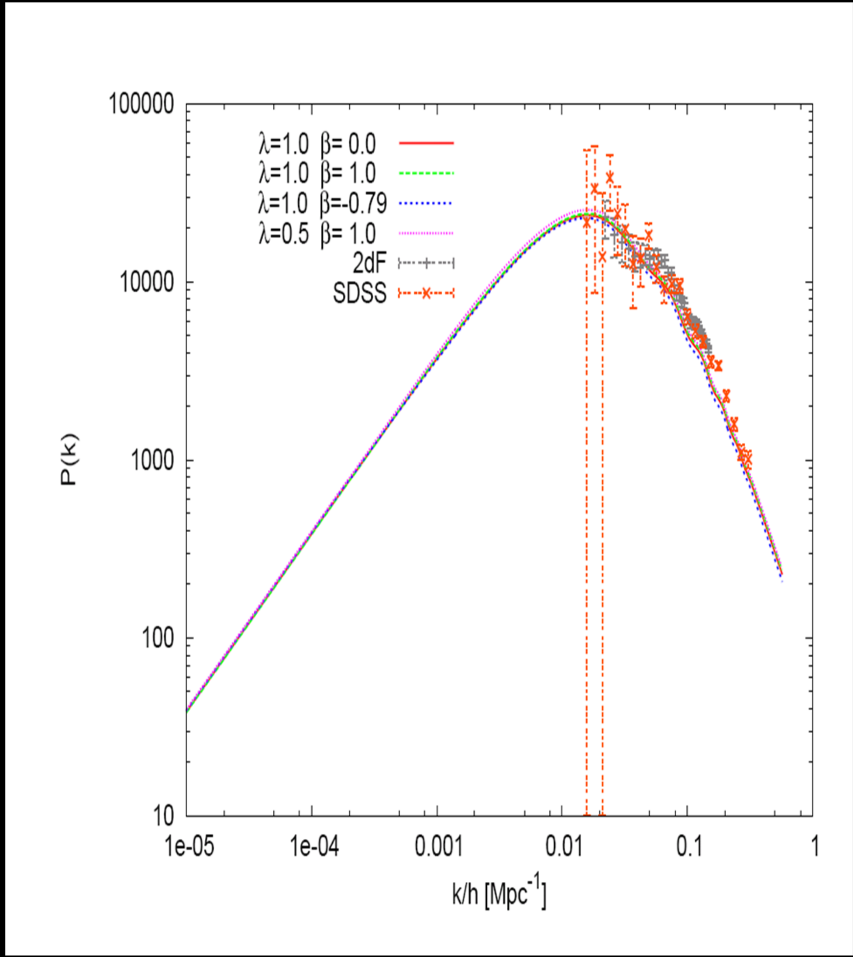
100
1000
l
l(2l+1)C_l/2π
λ=1.0 β= 0.0
λ=1.0 β= 1.0
λ=1.0 β=-0.79
λ=0.5 β= 1.0
WMAP 3 year data

Mν=0.3eV

Power-spectrum (LSS)

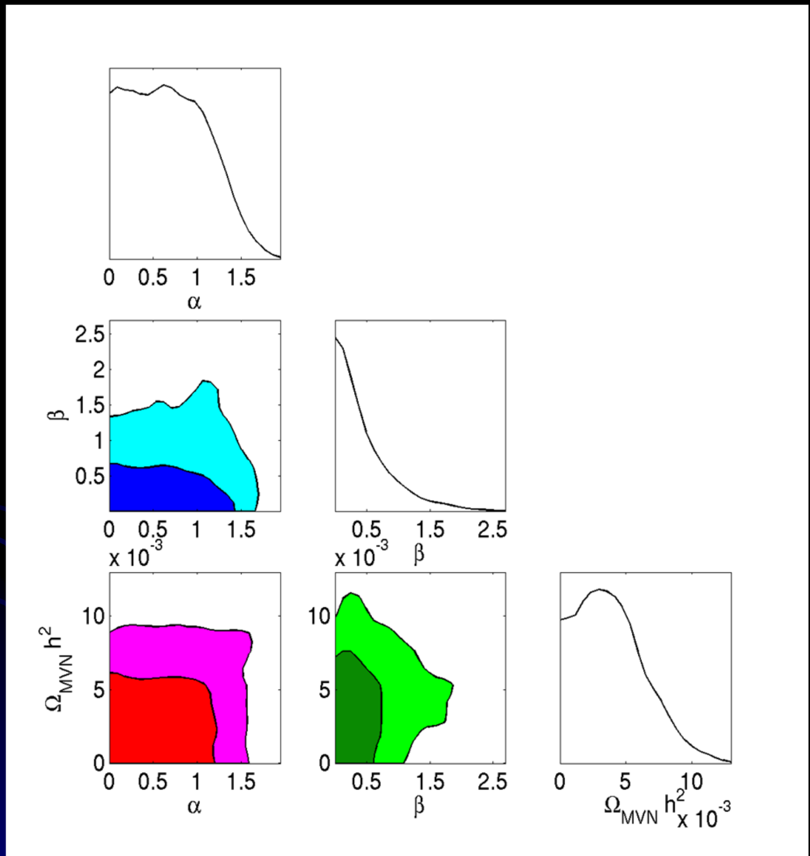


Mν=0.9 eV

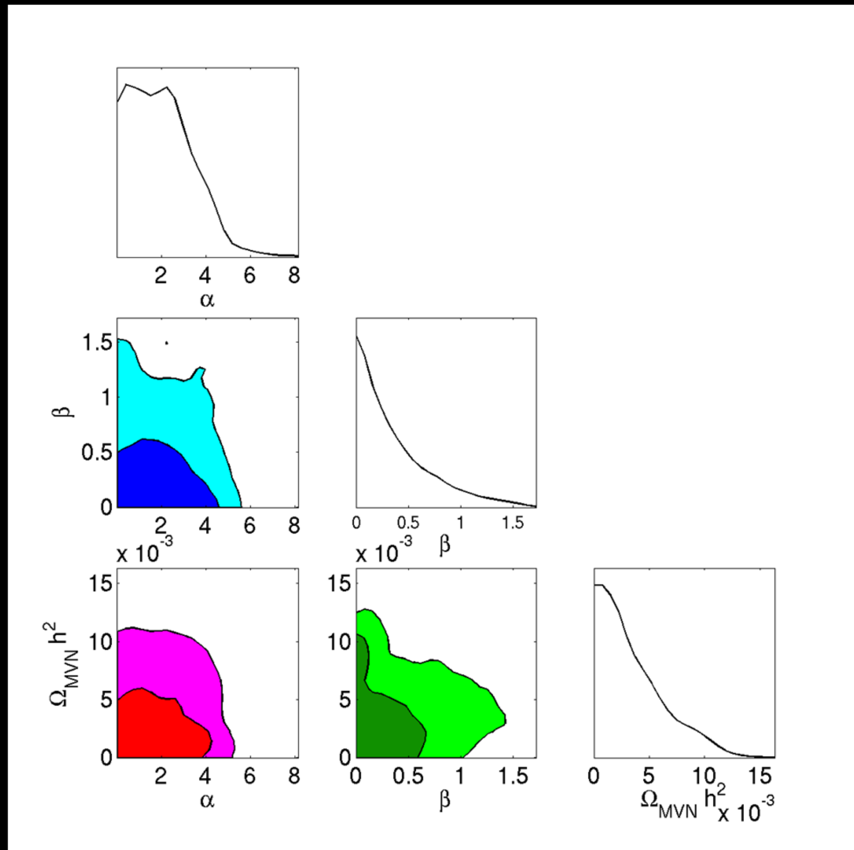


Mν=0.3 eV

Constraints from Observations



$$V(\phi) = V_0 e^{-\alpha\phi}$$

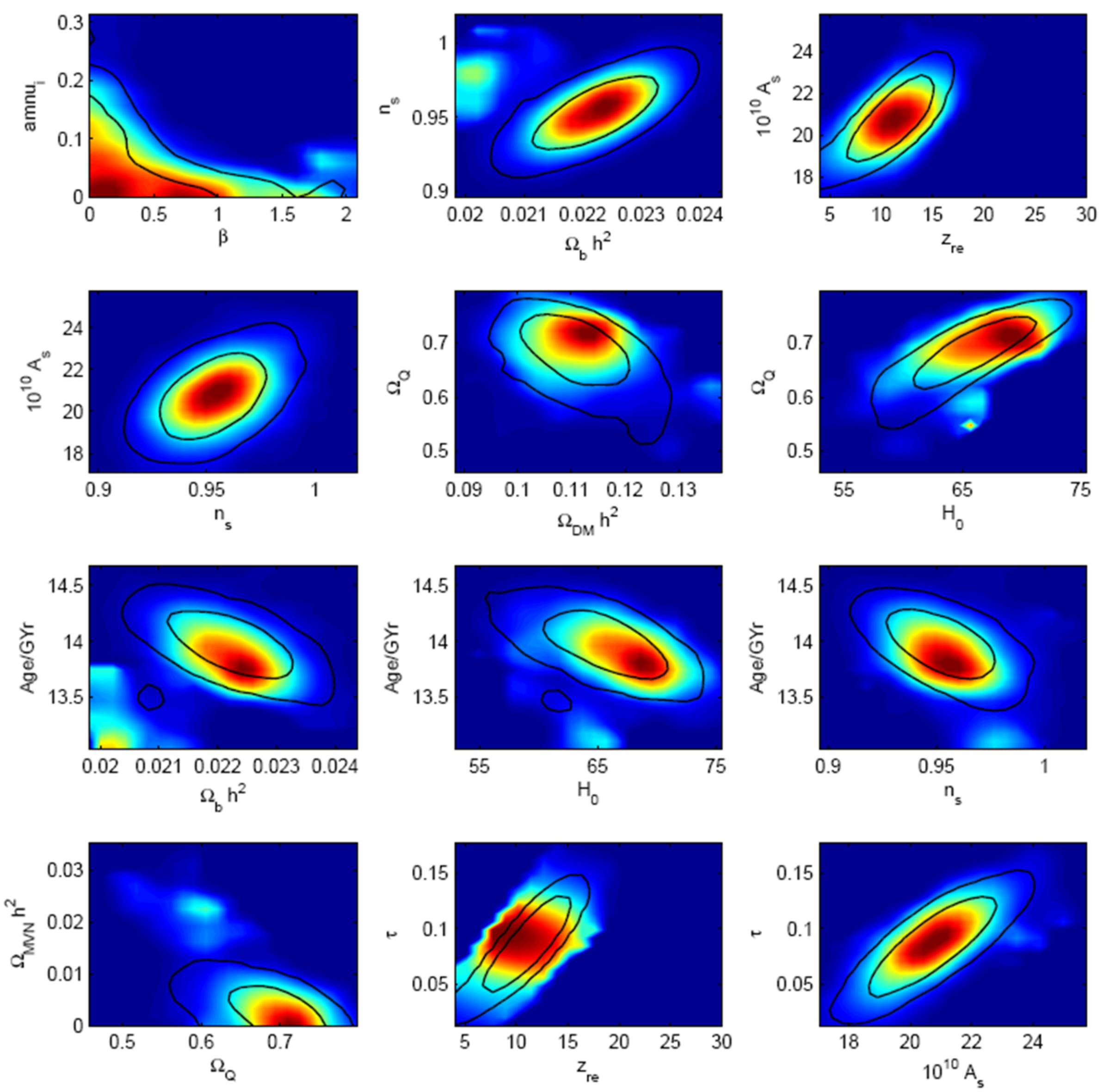


$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

	Exponential	Potential	Inverse-Power	Potential
Quantities	Means	1σ	Means	1σ
$\Omega_B h^2(10^2)$	2.21 ± 0.07	$2.15 - 2.28$	2.21 ± 0.07	$2.15 - 2.28$
$\Omega_{CDM} h^2(10^2)$	11.10 ± 0.63	$10.48 - 11.72$	11.10 ± 0.62	$10.52 - 11.68$
H_0	65.61 ± 3.26	$62.37 - 68.78$	65.97 ± 3.61	$62.30 - 69.37$
Z_{re}	11.07 ± 2.44	$10.07 - 12.35$	10.87 ± 2.58	$9.81 - 12.15$
α	0.70 ± 0.42	< 0.92	2.08 ± 1.35	< 2.63
β	0.50 ± 0.48	< 0.58	0.38 ± 0.35	< 0.46
$M_{\nu 0}(eV)$	0.047 ± 0.046	< 0.055	0.057 ± 0.070	< 0.051
n_s	0.95 ± 0.02	$0.94 - 0.97$	0.95 ± 0.02	$0.94 - 0.97$
$A_s(10^{10})$	20.72 ± 1.24	$19.47 - 21.95$	20.66 ± 1.31	$19.38 - 21.92$
$\Omega_Q(10^2)$	68.22 ± 4.17	$64.38 - 72.08$	68.54 ± 4.81	$64.02 - 72.94$
$Age/Gyrs$	13.69 ± 0.19	$13.77 - 14.15$	13.95 ± 0.20	$13.76 - 14.15$
$\Omega_{MVN} h^2(10^2)$	0.38 ± 0.25	< 0.48	0.36 ± 0.29	< 0.44
τ	0.09 ± 0.03	$0.06 - 0.11$	0.08 ± 0.03	$0.05 - 0.11$

Table 3: Global Fit analysis data using usual choice of potentials and coupling: $V(\phi) = V_0 e^{-\alpha\phi}$, $M^{4+\alpha}/\phi^\alpha$ and $m_\nu(\phi) = M_{\nu 0} e^{\beta\phi}$

Neutrino mass Bound: $M_\nu < 0.87$ eV @ 95 % C.L.



amnu_i
β
n_s
Ω_b h^2
10^10 A_s
z_re
Ω_Q
Ω_DM h^2
H_0
Age/GYr
Ω_MVN h^2
τ

Summary: Neutrino Mass Bounds in Interacting Neutrino DE Model

Without Ly-alpha Forest data (only 2dFGRS + HST + WMAP3)

- Omega_nu h^2 < 0.0044 ; 0.0095 (inverse power-law potential)
- < 0.0048 ; 0.0090 (sugra type potential)
- < 0.0048 ; 0.0084 (exponential type potential)

provides the total neutrino mass bounds

M_nu < 0.45 eV (68 % C.L.)
< 0.87 eV (95 % C.L.)

Including Ly-alpah Forest data

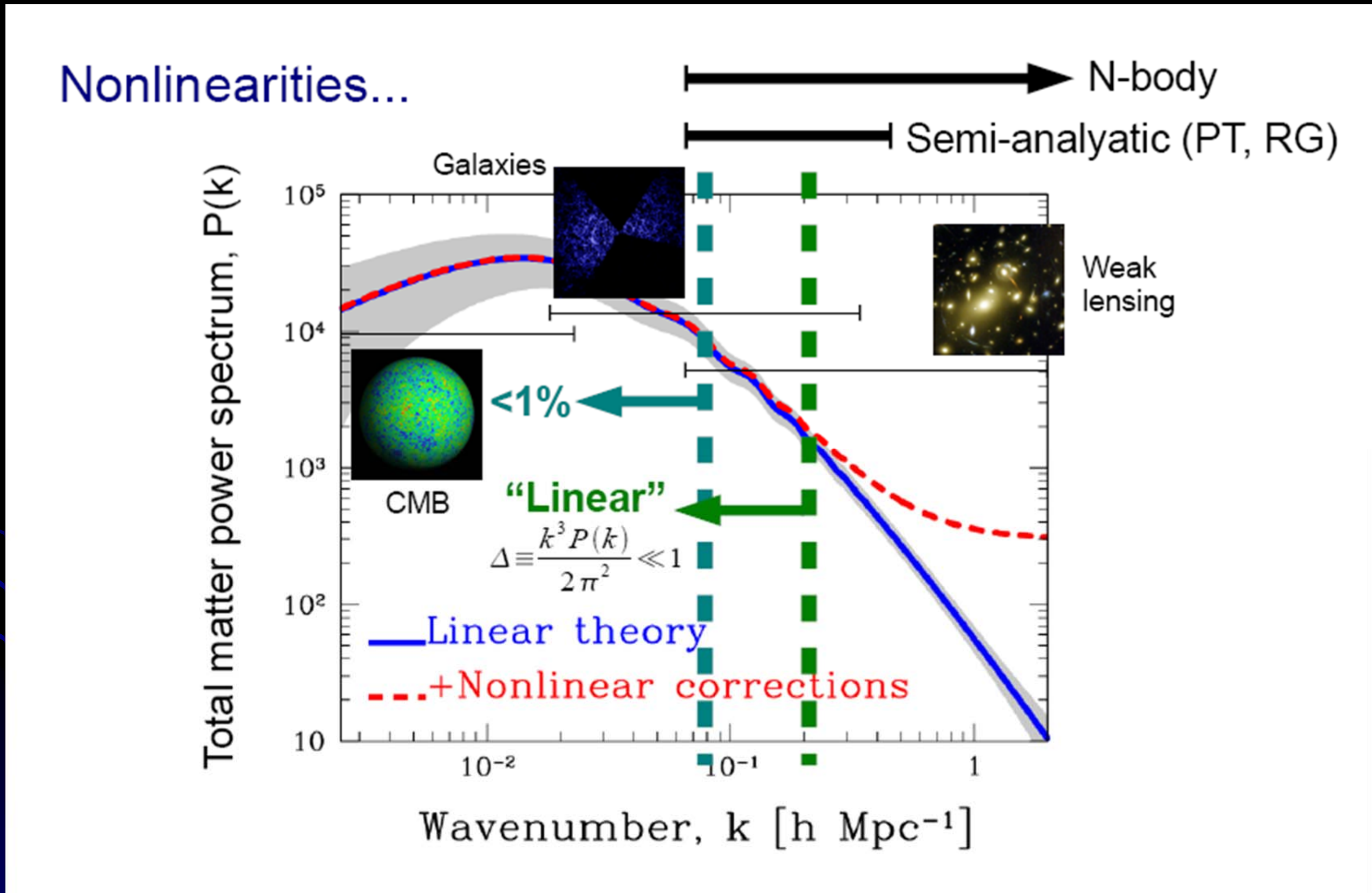
Omega_nu h^2 < 0.0018; 0.0046 (sugra type potential)

corresponds to

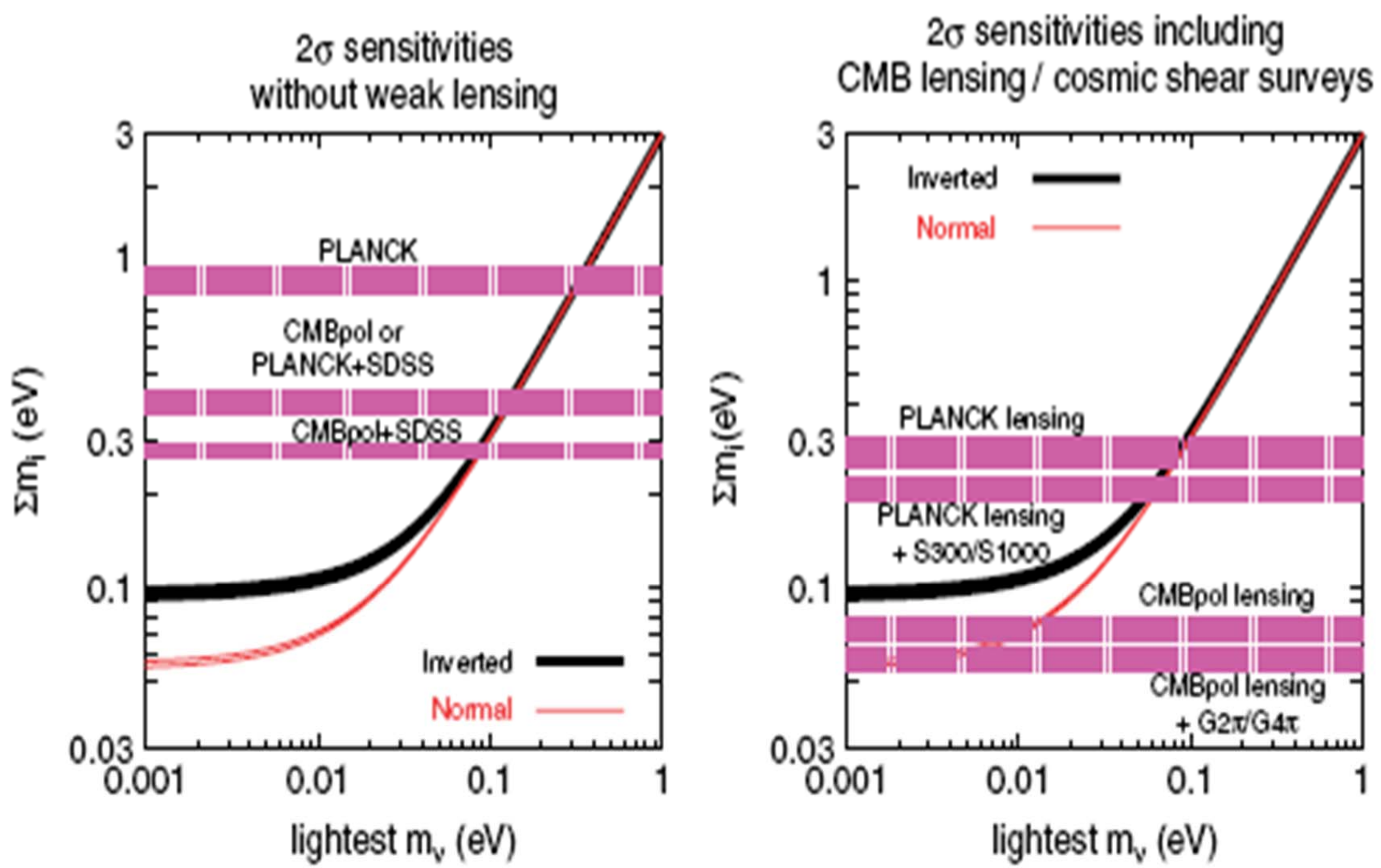
M_nu < 0.17 eV (68 % C.L.)
< 0.43 eV (95 % C.L.)

We have weaker bounds in the interacting DE models

Nonlinear Effects



Future Prospects from Astrophysical Observations



Summary

- LCDM model provides
  M_nu < 0.6-0.7 eV (LSS + CMB +BAO)
  < 0.2-0.3 eV (including Lya data)

- Interacting Neutrino Dark-Energy Model provides more weaker bounds:
  M_nu < 0.8-0.9 eV (LSS + CMB)
  < 0.4-0.5 eV (including Lya data)
- Lya-forest data includes the uncertainty from
  - continuum errors
  - unidentified metal lines
  - noise

Summary of Methods to Obtain Neutrino Masses

Single beta decay	$\Sigma_i\, m_i^2\, \lvert U_{ei}\rvert^2$	Sensitivity 0.2 eV
Double beta decay	$m_{\beta\beta} = \lvert \Sigma_i\, m_i\, \lvert U_{ei}\rvert^2\, \varepsilon_i \rvert$ ε_i = Majorana phases	Sensitivity 0.01 eV
Neutrino oscillations	$\delta m^2 = m_1^2 - m_2^2$	Observed ~ 10^{-5} eV^2
Cosmology	$\Omega_\nu \rightarrow \Sigma_i\, m_i$	Observed ~ 0.1 eV

Only double beta decay is sensitive to Majorana nature.

Thanks

For

your attention!

Backup Slides

Cosmological parameters

- Omega_c : fraction of the dark-matter density
- Omega_b: fraction of the baryon matter density
- Theta: the (approx) sound horizon to the angular diameter distance
- tau: optical depth
- n_s : scale spectral index
- Ln[10^10 As] : primordial superhorizon power in the curvature perturbation on 0.05 Mpc^-1 scale

Theoretical issue: Adiabatic Instability problem:

Afshordi et al. 2005

$m_{eff}^2 = d^2V_{eff} / d\phi^2$

- $m_{eff} \gg H$ (Chameleon DE models)

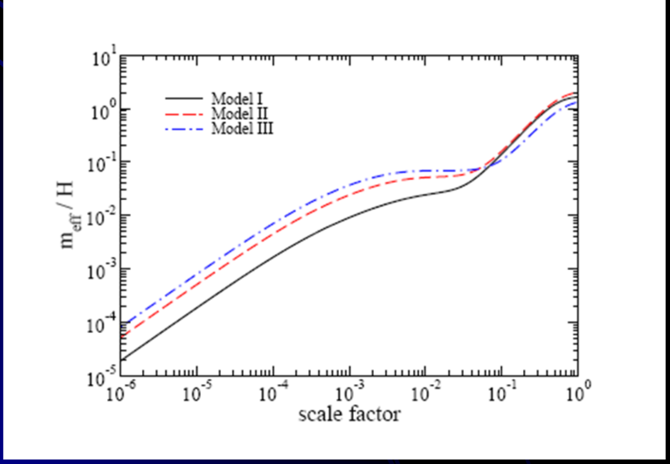
- Gravitational collapse

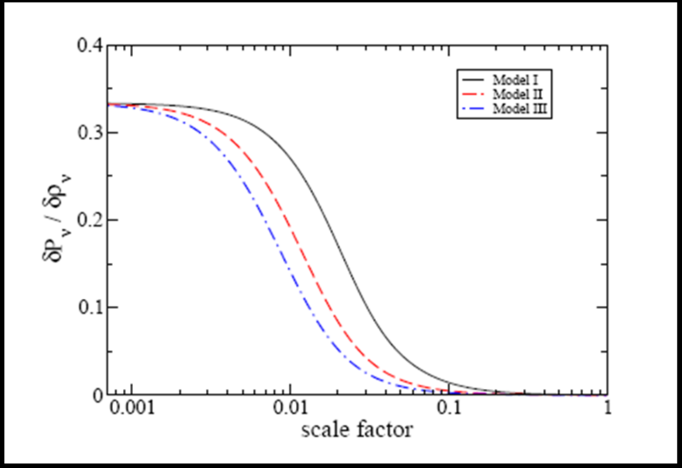
- Kaplan, Nelson, Weiner 2004
- Khoury et al. 2004
- Zhao, Xia, X.M Zhang 2006

- $m_{eff} < H$ (Slow-rolling Condition)

- Always positive sound velocity
- No adiabatic instability

- Brookfield et al,. 2006
- YYK and Ichiki, 2007, 2008





Energy Density vs scale factor

yyk and ichiki, JHEP 0806,085 2008

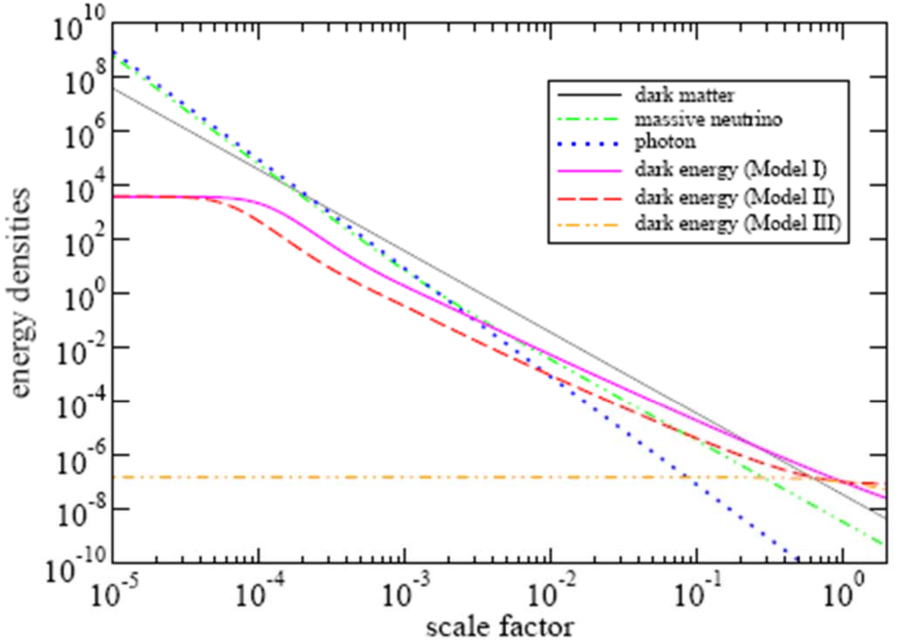
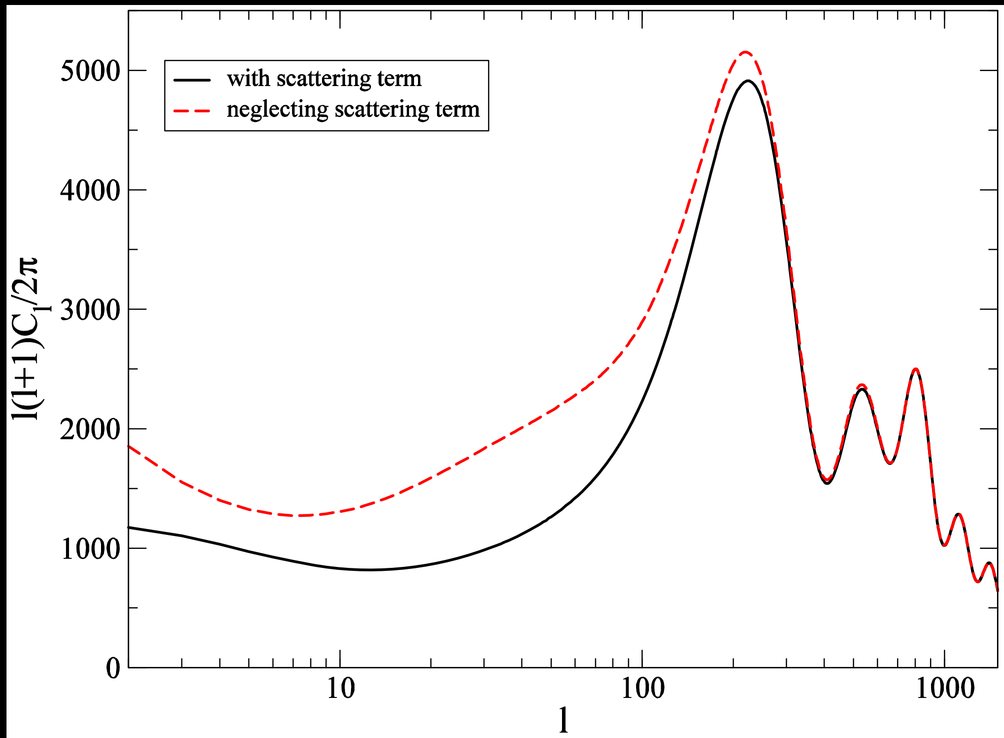
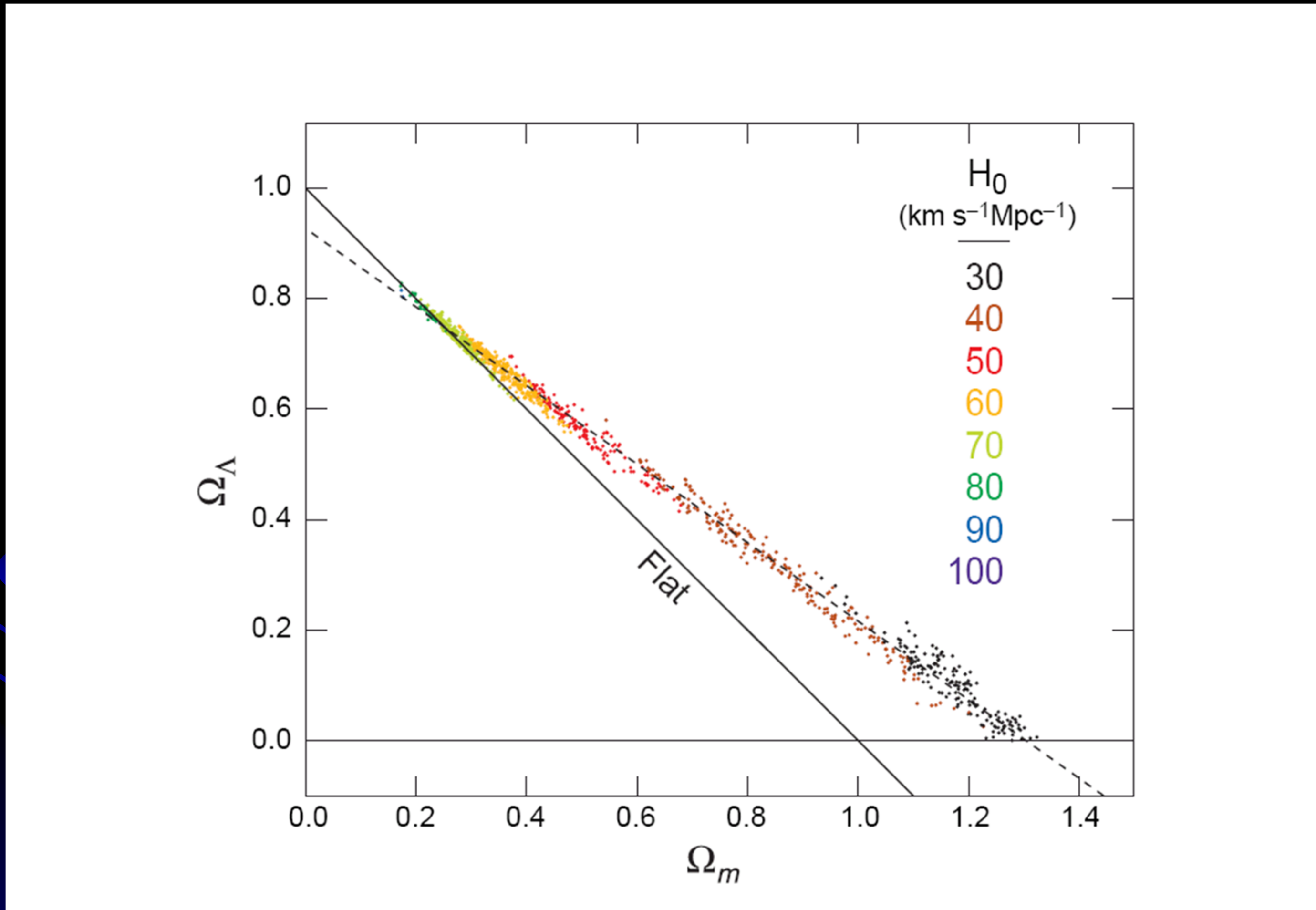


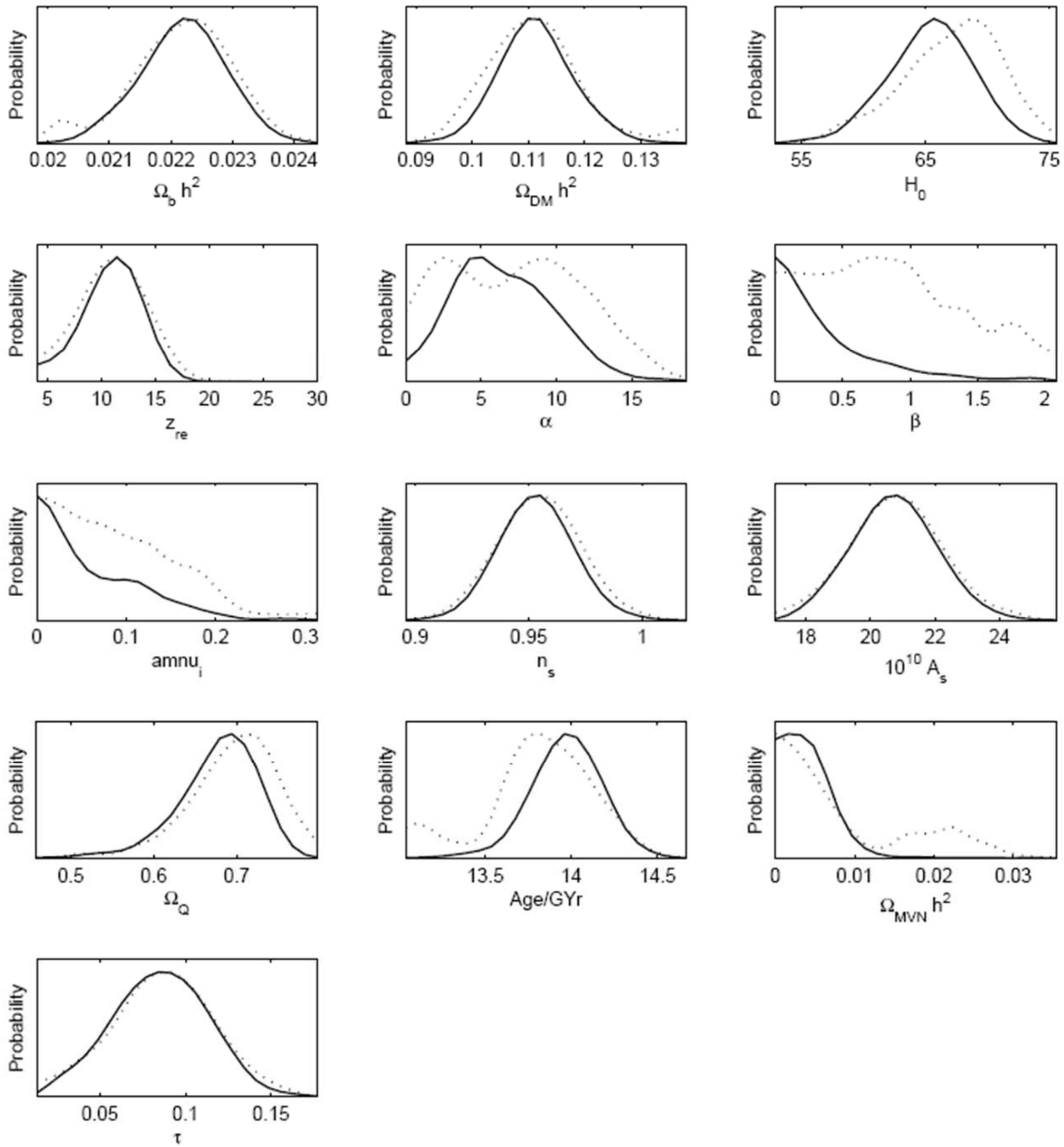
Figure 2: Examples of the evolution of energy density in quintessence and the background fields as indicated. Model parameters taken to plot this figure are $\alpha = 10, 10, 1$ for model I, II, III, respectively. The other parameters for the dark energy are fixed so that the energy densities in three types of dark energy should be the same at present.

The impact of Scattering term:



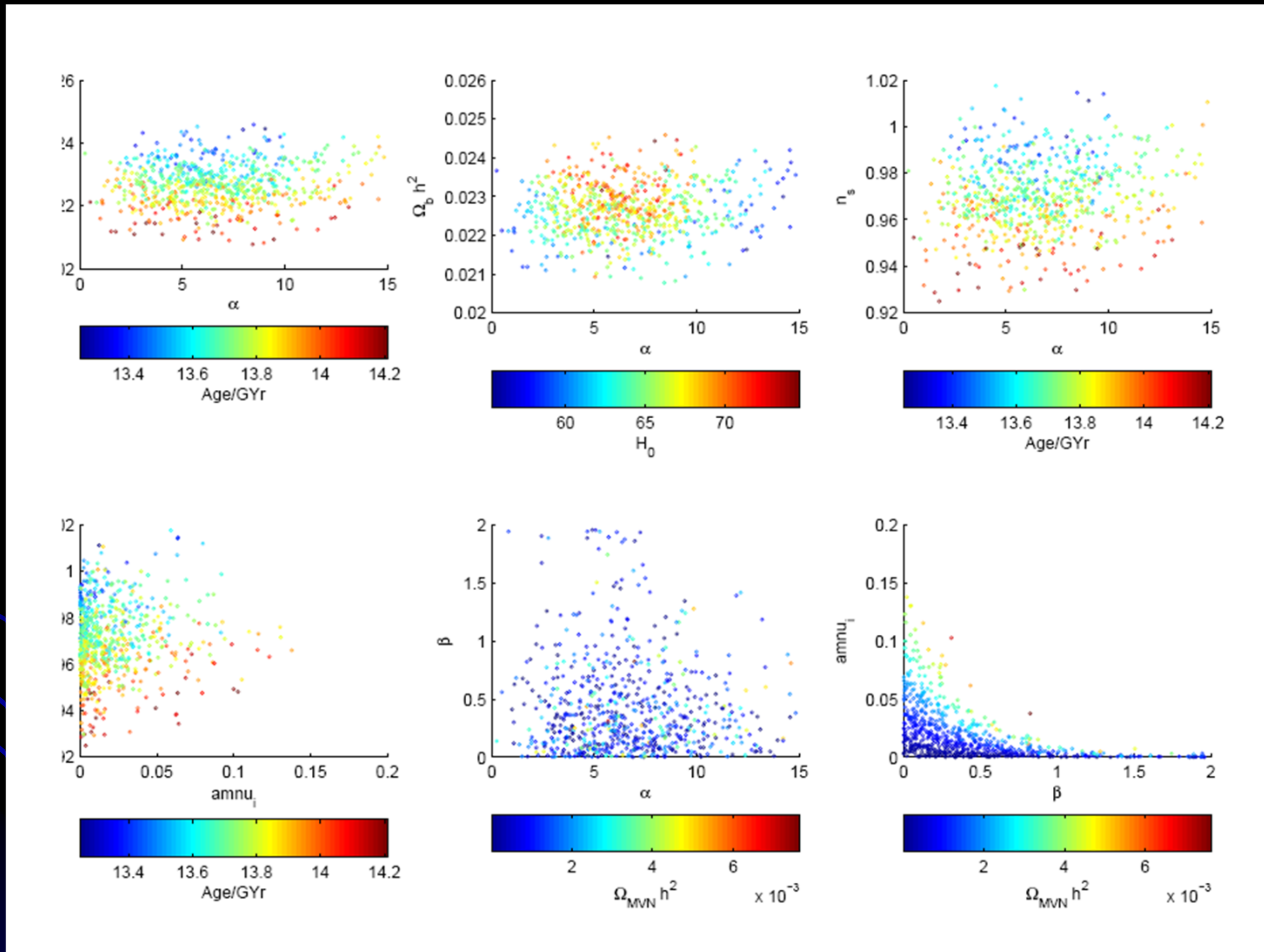
WMAP3 data on Ho vs Ω



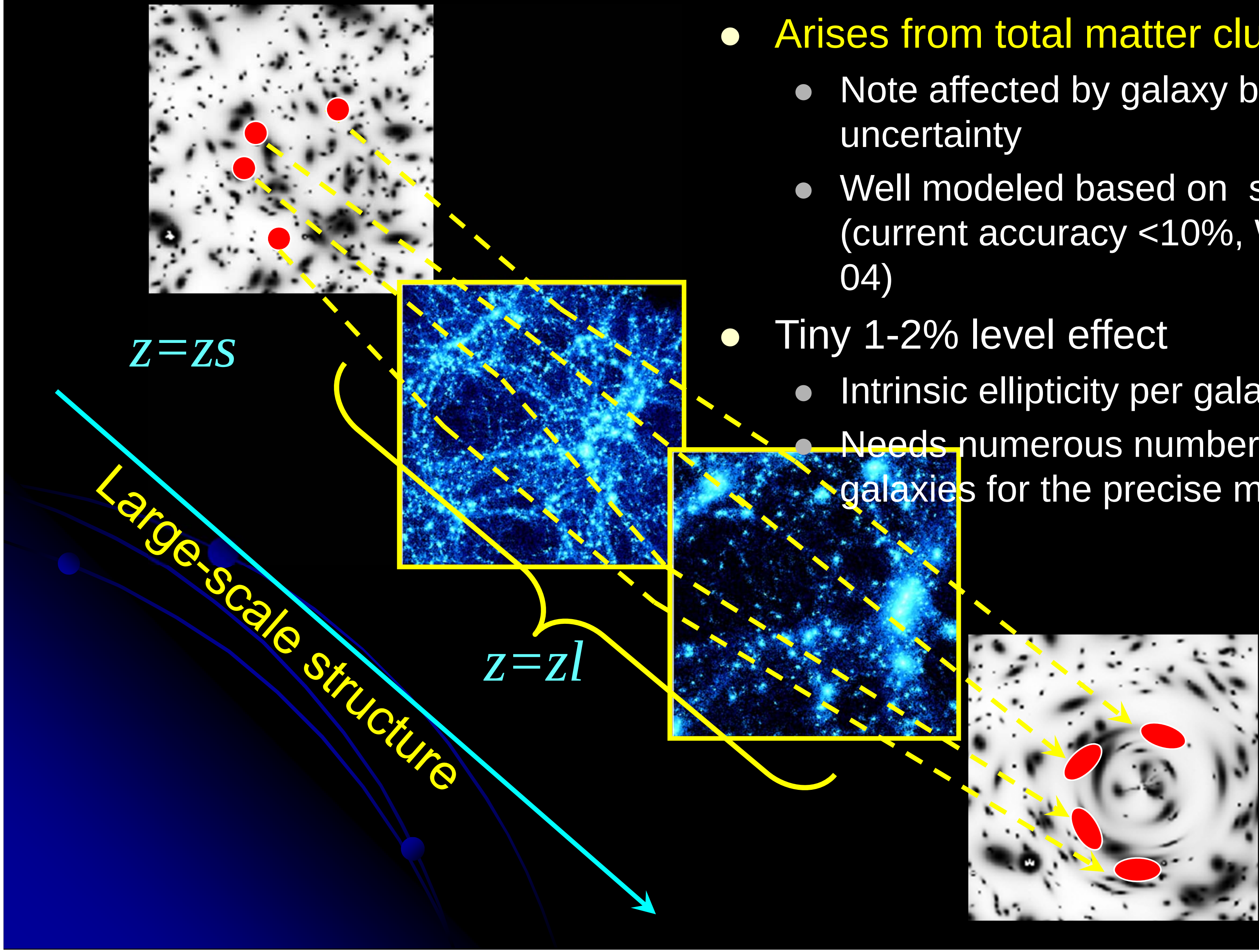


Probability
0.02 0.021 0.022 0.023 0.024
$\Omega_b h^2$
0.09 0.1 0.11 0.12 0.13
$\Omega_{DM} h^2$
55 65 75
H_0
5 10 15 20 25 30
z_{re}
0 5 10 15
α
0 0.5 1 1.5 2
β
0 0.1 0.2 0.3
$amnu_i$
0.9 0.95 1
n_s
18 20 22 24
$10^{10} A_s$
0.5 0.6 0.7
Ω_Q
13.5 14 14.5
Age/GYr
0 0.01 0.02 0.03
$\Omega_{MVN} h^2$
0.05 0.1 0.15
τ

Joint 3-dimensional intercorrelations between Cosmological Parameters and Model Parameters



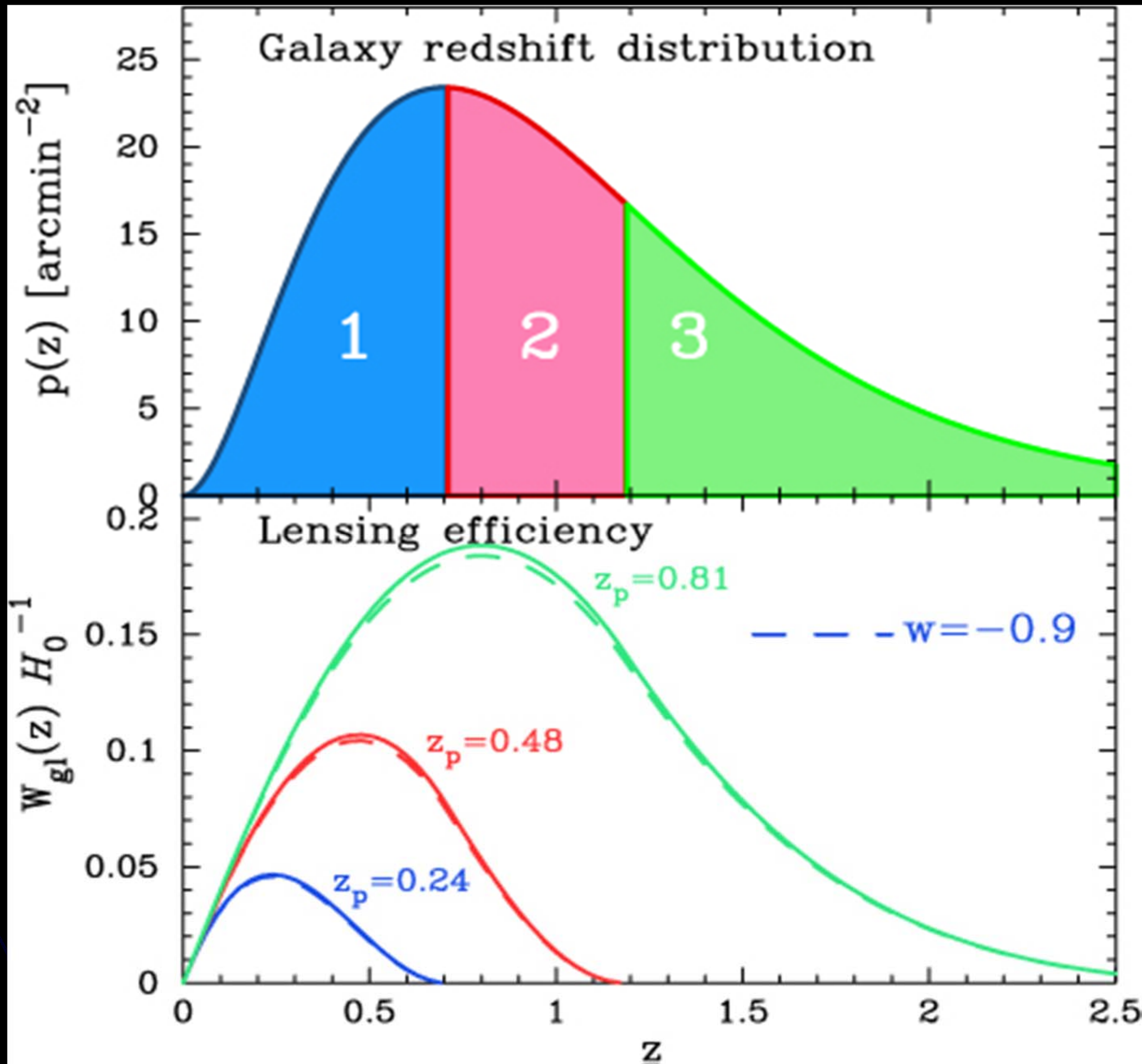
Cosmological weak lensing



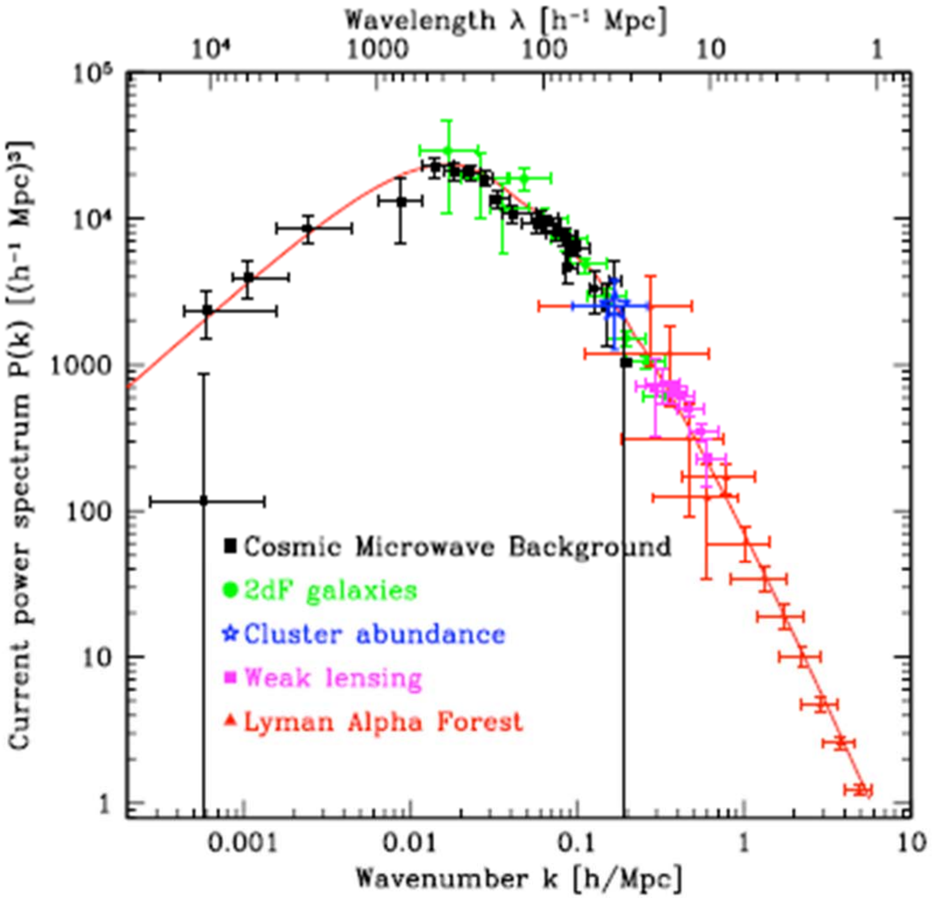
- Arises from total matter clustering
  - Note affected by galaxy bias uncertainty
  - Well modeled based on simulations (current accuracy <10%, White & Vale 04)
- Tiny 1-2% level effect
  - Intrinsic ellipticity per galaxy, ~30%
  - Needs numerous number (10^8) of galaxies for the precise measurement

Weak Lensing Tomography- Method

- Subdivide source galaxies into several bins based on photo-z derived from multi-color imaging
- $<z_i>$ in each bin needs accuracy of ~0.1%
- Adds some ``depth" information to lensing – improve cosmological parameters (including DE).



Matter Power Spectrum from the Lyman alpha Forest



<u>IGM smoothed</u> by:

- Jeans length
- thermal motion
- pec. velocities

<u>spurious structure</u> from:

- continuum errors
- unidentified metal lines
- noise

- equation of state (?)
- fluctuating rad. field (?)
- galactic winds (?)

Questions :

- How can we test mass-varying neutrino model in Exp. ?

  --- by the detection of the neutrino mass variation in space via neutrino oscillations.

  Barger et al., M. Cirelli et al., 2005

  --- by the measurement of the time delay of the neutrino emitted from the short gamma ray bursts.

  X.M. Zhang et al. yyK in preparing

- How much this model can be constrained from, BBN, CMB, Matter power spectrum observations ?

  Ichiki and YYK, 2008, 2010

Solar mass-varying neutrino oscillation

V.Barger et al: hep-ph/0502196;PRL2005
M.Cirelli et al: hep-ph/0503028

- The evolution eq. in the two-neutrinos framework are:

$$i\frac{d}{dr}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{1}{2E_\nu}\left[U\begin{pmatrix}(m_1-M_1(r))^2 & M_3(r)^2\\ M_3(r)^2 & (m_2-M_2(r))^2\end{pmatrix}U^\dagger + \begin{pmatrix}A(r) & 0\\ 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

- ν_e-e forward scattering amplitude:

$$A(r) = 2\sqrt{2}\,G_F n_e(r) E_\nu = 1.52\times10^{-7}\mathrm{eV}^2\, n_e(r)\, E_\nu\,(\mathrm{MeV})$$

- Model dependence in the matter profiles:

$$M_i(r) = \mu_i\left(\frac{n_e(r)}{n_e^0}\right)^k$$

- k parameterize the dependence of the neutrino mass on n_e
- μ_i is the neutrino mass shift at the point of neutrino production.

MaVaN results: