Bose-Einstein Condensation of Dark Matter Axions

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7th Patras Workshop on Axions, WIMPs and WISPs

Mykonos (GR), 26 June – 1 July 2011

Outline

- Bose-Einstein condensation of dark matter axions (axions are different)

- the inner caustics of galactic halos

  (axions are better)

- axions and cosmological parameters

Dark matter candidates

		axion	WIMP	sterile ν
mass	m	$10^{-5}\ \frac{\mathrm{eV}}{c^2}$	$100\frac{\mathrm{GeV}}{c^2}$	$10\frac{\mathrm{keV}}{c^2}$
velocity dispersion	δv	$10^{-17}c$	$10^{-12}c$	$10^{-8}c$
coherence length	$\ell = \frac{\hbar}{m\ \delta v}$	$10^{17}\mathrm{cm}$	$10^{-5}\mathrm{cm}$	$10^{-1}\mathrm{cm}$

QFT has two classical limits:

limit of point particles (WIMPs, ...)

$$\hbar \to 0 \qquad\qquad \omega, \vec{k} \to \infty$$

$$E = \hbar\omega \;\; \text{and} \;\; \vec{p} = \hbar\vec{k} \;\; \text{fixed}$$

limit of classical fields (axions)

$$\hbar \to 0 \qquad\qquad N \to \infty$$

$$E = N\hbar\omega \;\; \text{and} \;\; \vec{p} = N\hbar\vec{k} \;\; \text{fixed}$$

Cold axion properties

- number density

$$n(t) \sim \frac{4 \cdot 10^{47}}{\text{cm}^3} \left(\frac{f_a}{10^{12}\ \text{GeV}} \right)^{\frac{5}{3}} \left(\frac{a(t_1)}{a(t)} \right)^3$$

- velocity dispersion

$$\delta \text{v}(t) \sim \frac{1}{m_a\, t_1} \frac{a(t_1)}{a(t)}$$

if decoupled

- phase space density

$$\mathcal{N} \sim n(t) \frac{(2\pi)^3}{\frac{4\pi}{3}(m_a\, \delta \text{v})^3} \sim 10^{61} \left(\frac{f_a}{10^{12}\ \text{GeV}} \right)^{\frac{8}{3}}$$

Bose-Einstein Condensation

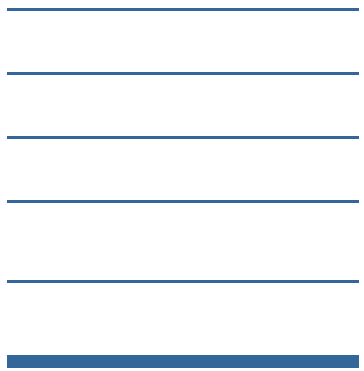
if identical bosonic particles
are highly condensed in phase space
and their total number is conserved
and they thermalize

then most of them go to the lowest energy
available state

why do they do that?

by yielding their energy to the non-condensed particles, the total entropy is increased.

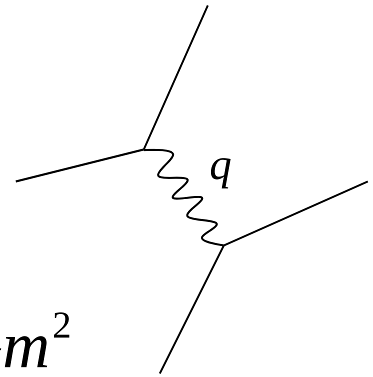
preBEC



BEC

Thermalization occurs due to gravitational interactions

PS + Q. Yang, PRL 103 (2009) 111301



$$\frac{Gm^2}{q^2}$$

$$\Gamma_g \sim 4\pi G n m^2 l^2 \quad \text{with } l = (m\,\delta \mathrm{v})^{-1}$$

$$\sim 5 \cdot 10^{-7} H(t_1) \left(\frac{f}{10^{12}\ \mathrm{GeV}} \right)^{\frac{2}{3}}$$

at time t_1

$$\Gamma_g(t)/H(t) \propto t\, a(t)^{-1} \propto a(t)$$

Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

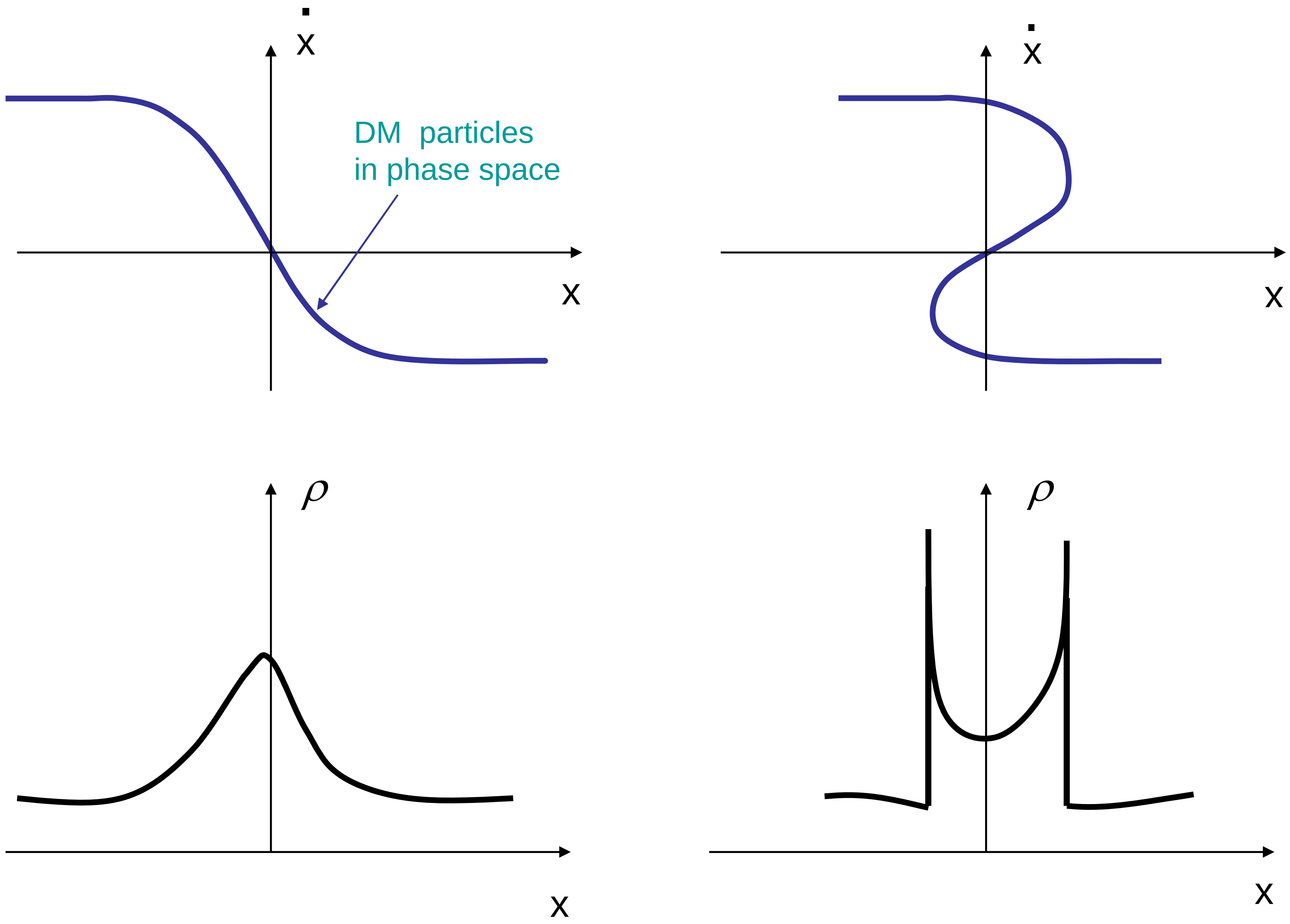
$$T_\gamma \sim 500 \text{ eV} \left(\frac{f}{10^{12} \text{ GeV}} \right)^{\frac{1}{2}}$$

After that

$$\delta \mathrm{v} \sim \frac{1}{m\,t}$$

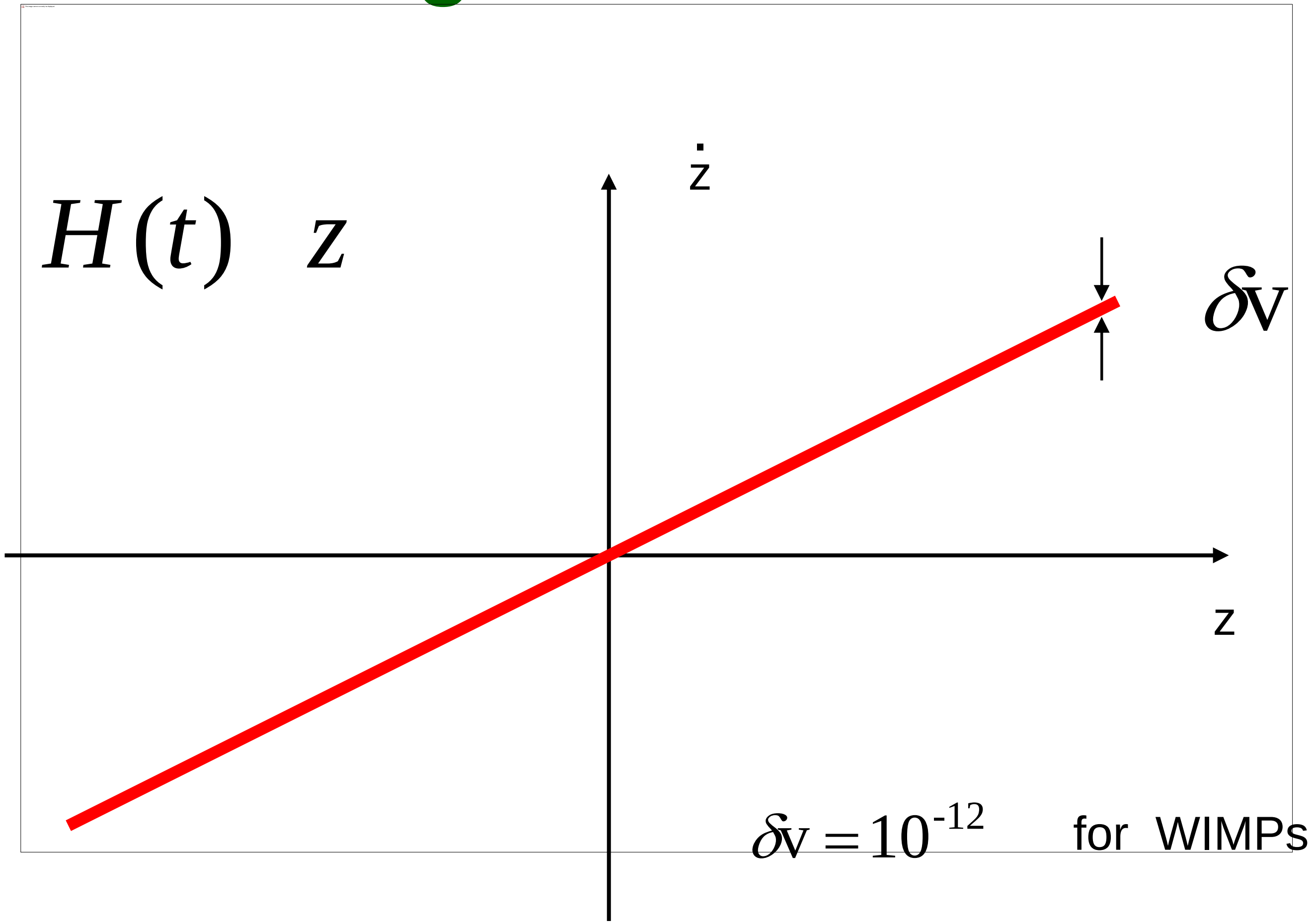
$$\Gamma_g(t) / H(t) \propto t^3 \, a(t)^{-3}$$

DM forms caustics in the non-linear regime



Phase space distribution of DM in a homogeneous universe

$$\dot{z} = H(t)\, z$$



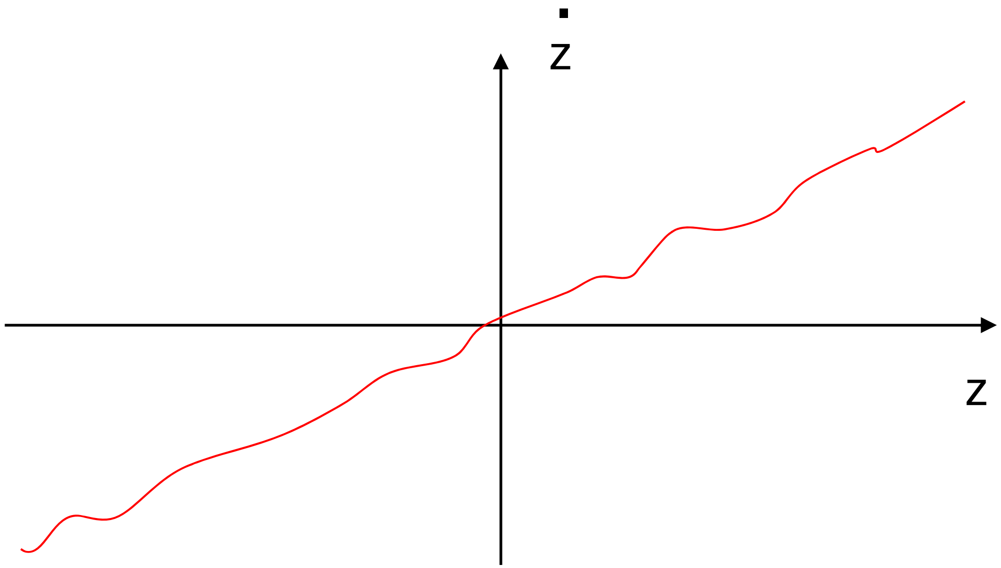
$\delta v = 10^{-12}$ for WIMPs

$\delta v = 10^{-17}$ for axions (preBEC)

$\delta v = 10^{-8}$ for sterile neutrinos

The dark matter particles lie on a 3-dimensional sheet in 6-dimensional phase space

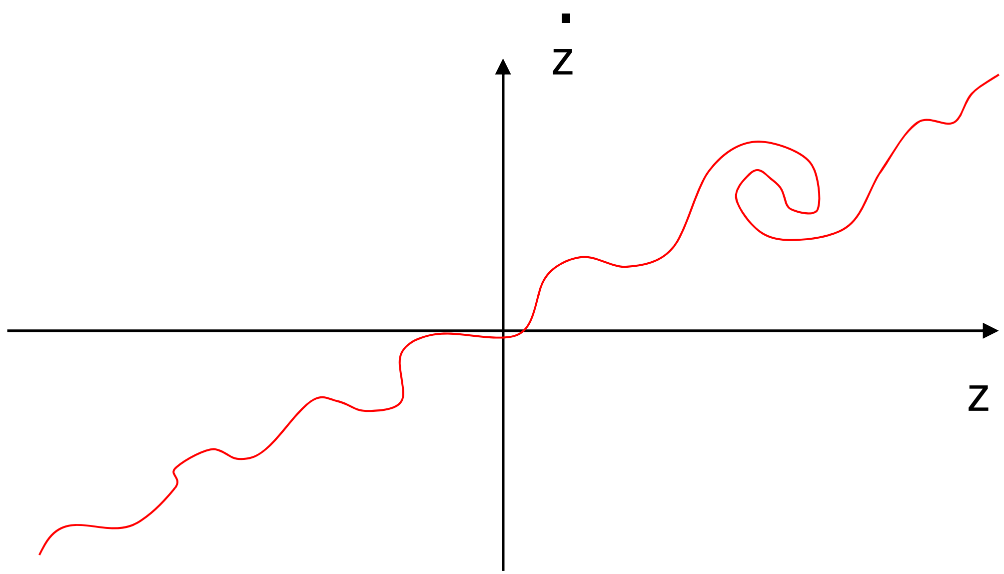
the physical density is the projection of the phase space sheet onto position space



$$\vec{v}(\vec{r},t) = H(t)\,\vec{r} + \Delta\vec{v}(\vec{r},t)$$

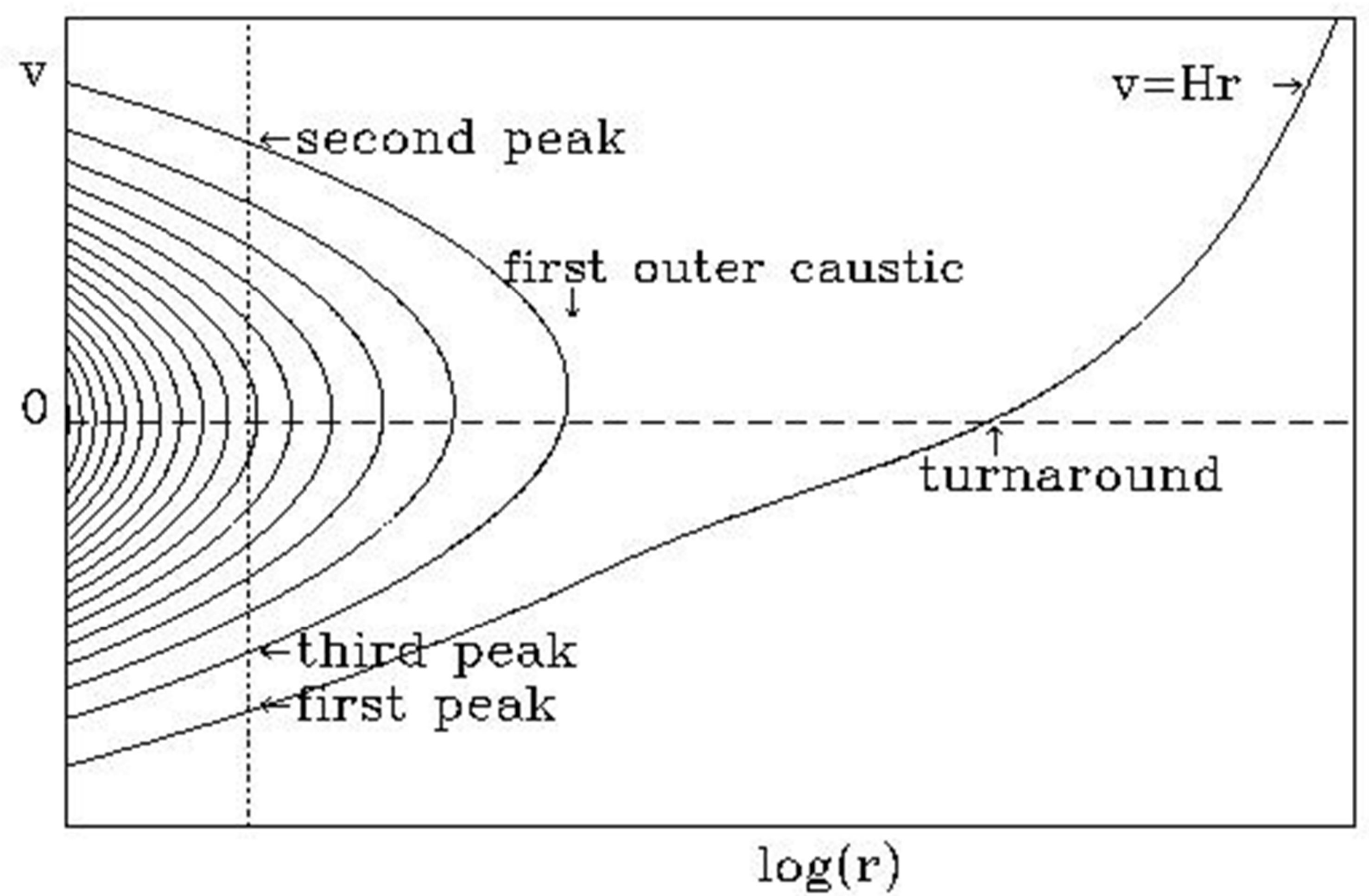
The cold dark matter particles lie on a 3-dimensional sheet in 6-dimensional phase space

the physical density is the projection of the phase space sheet onto position space



$$\vec{v}(\vec{r},t) = H(t)\,\vec{r} + \Delta\vec{v}(\vec{r},t)$$

Phase space structure of spherically symmetric halos



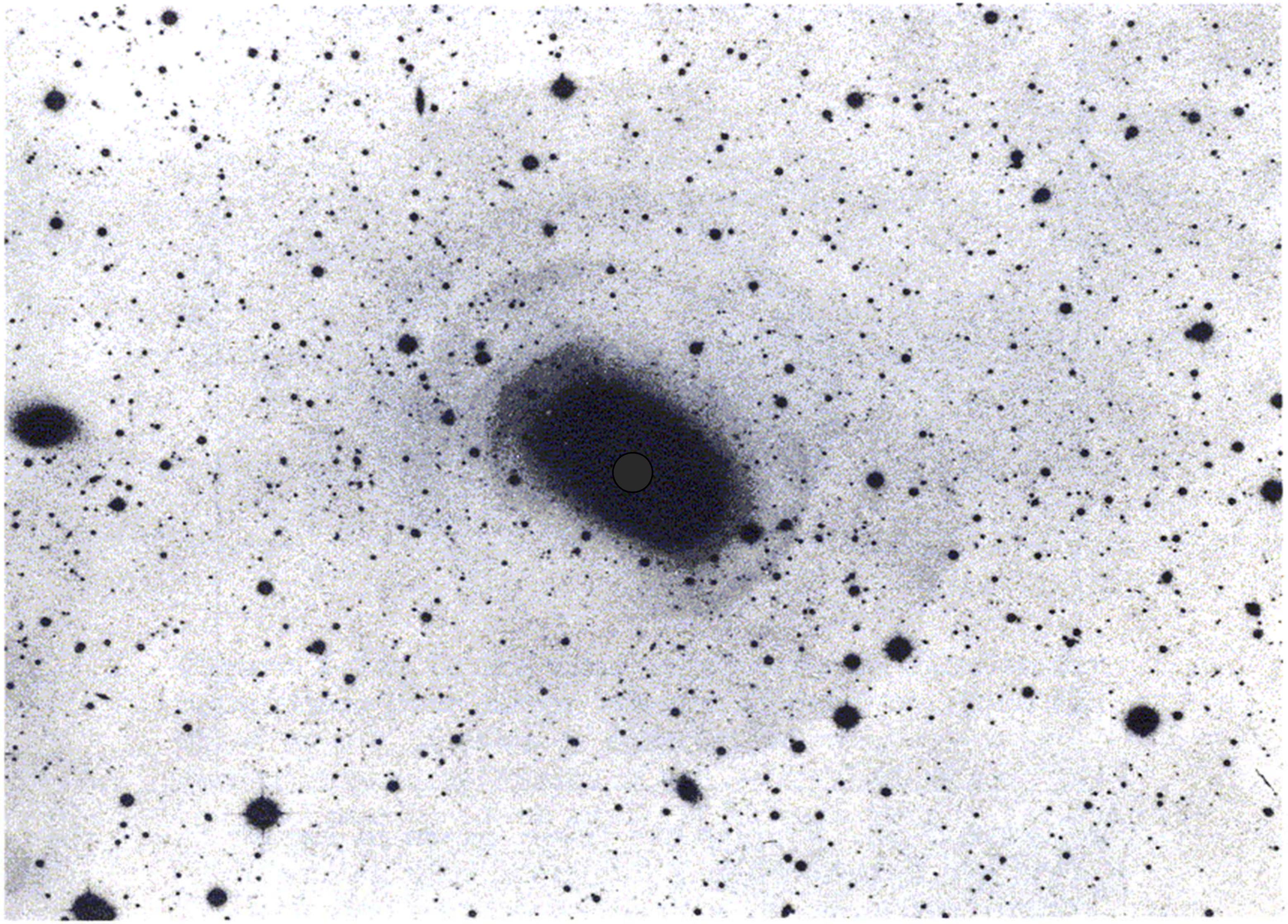


Figure 7-22. The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.

(from Binney and Tremaine's book)

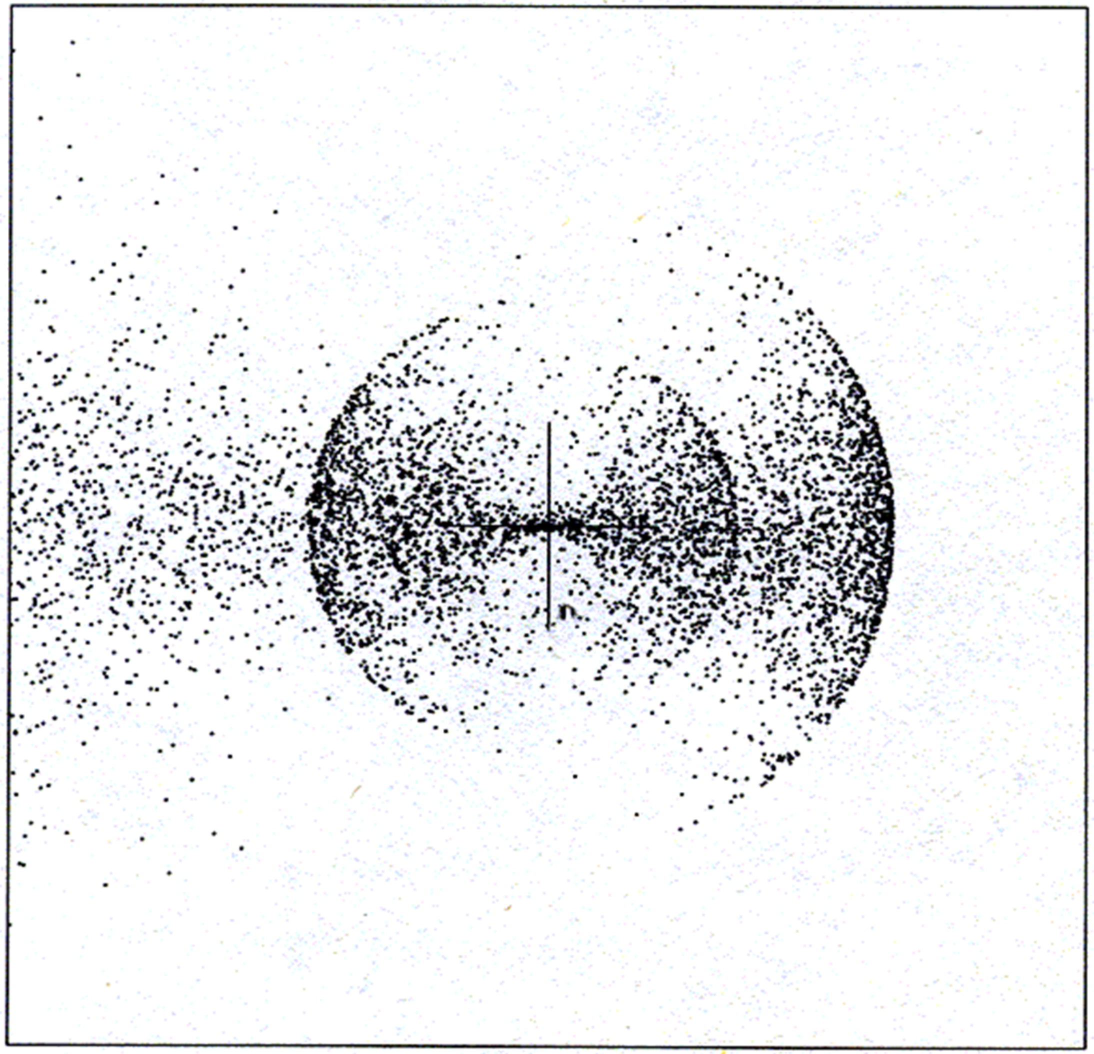
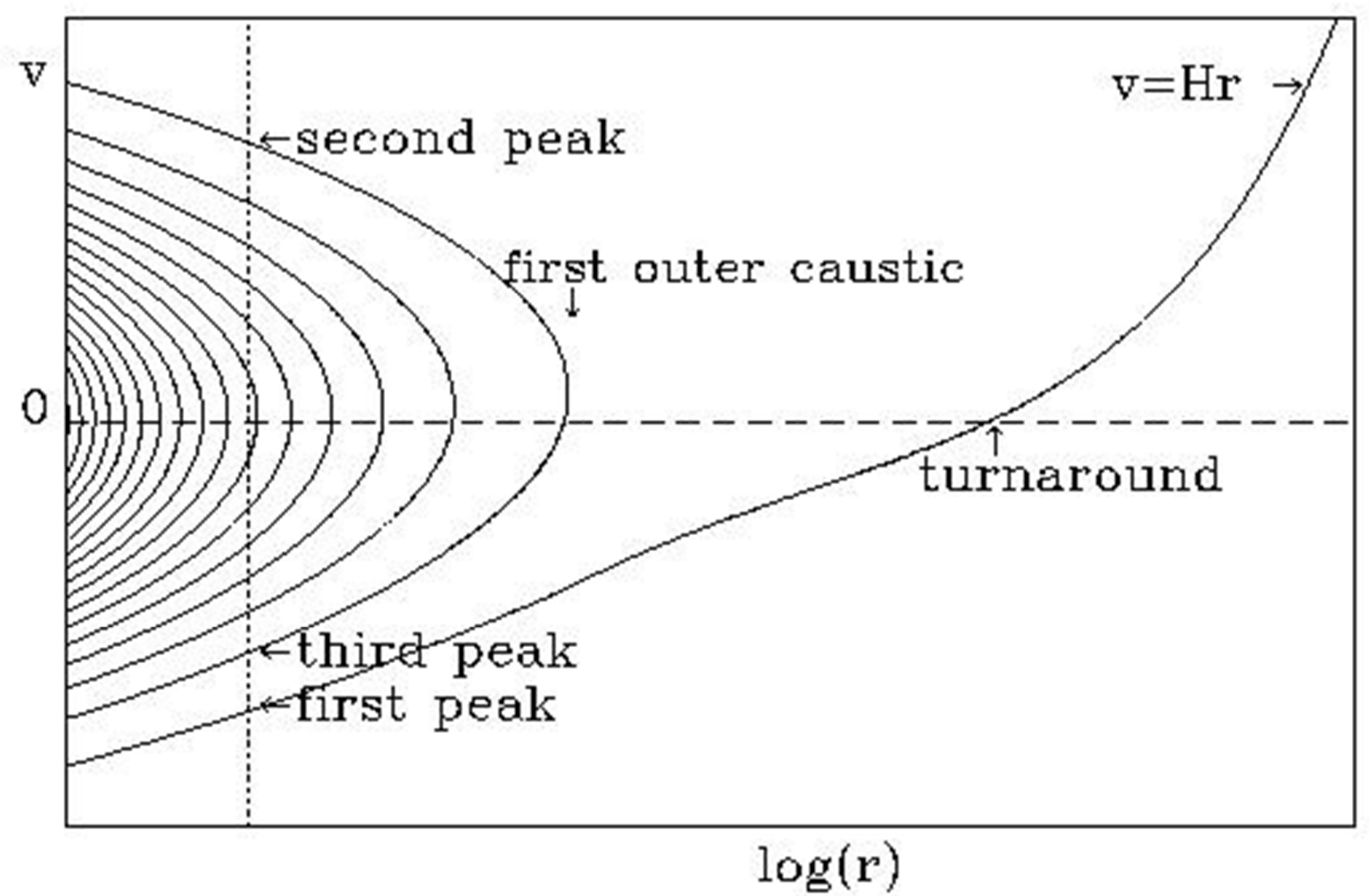


Figure 7-23. Ripples like those shown in Figure 7-22 are formed when a numerical disk galaxy is tidally disrupted by a fixed galaxy-like potential. (See Hernquist & Quinn 1987.)

Phase space structure of spherically symmetric halos



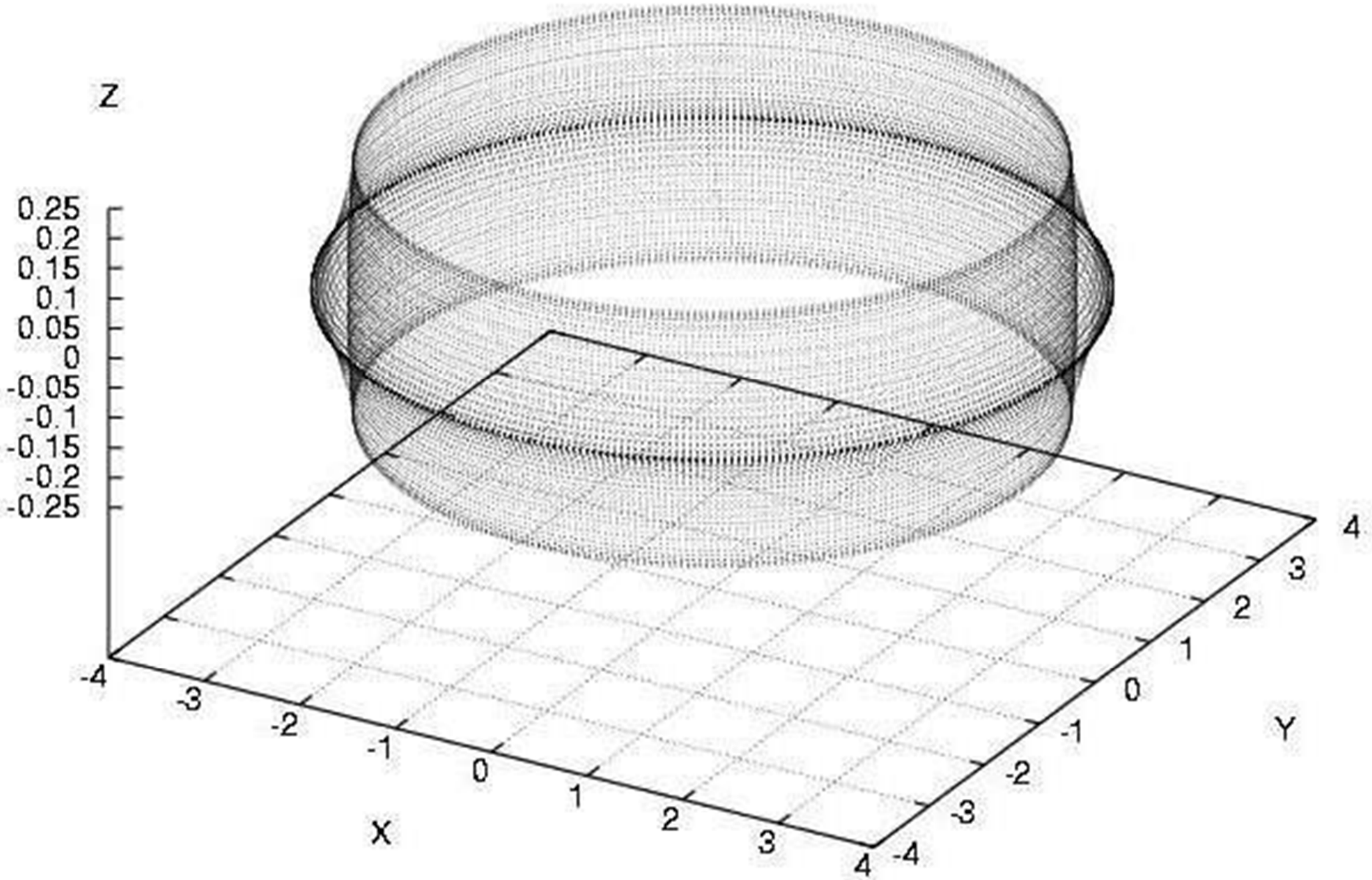
Galactic halos have inner caustics as well as outer caustics.

If the initial velocity field is dominated by net overall rotation, the inner caustic is a ‘tricusp ring’.

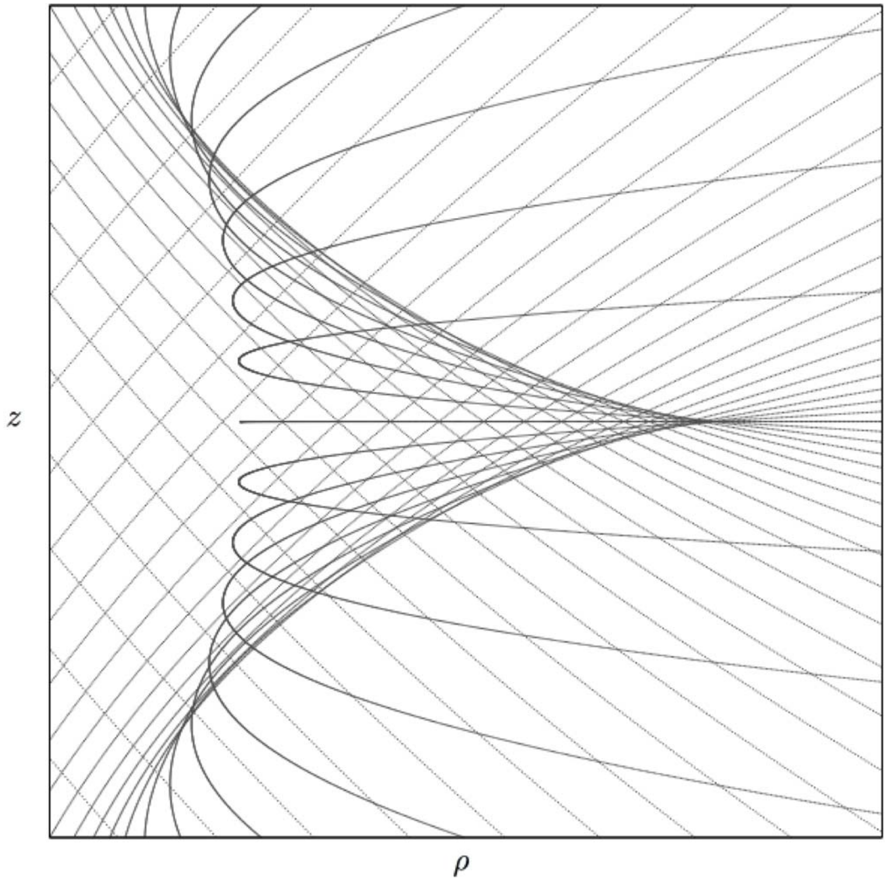
If the initial velocity field is irrotational, the inner caustic has a ‘tent-like’ structure.

(Arvind Natarajan and PS, 2005).

simulations by Arvind Natarajan

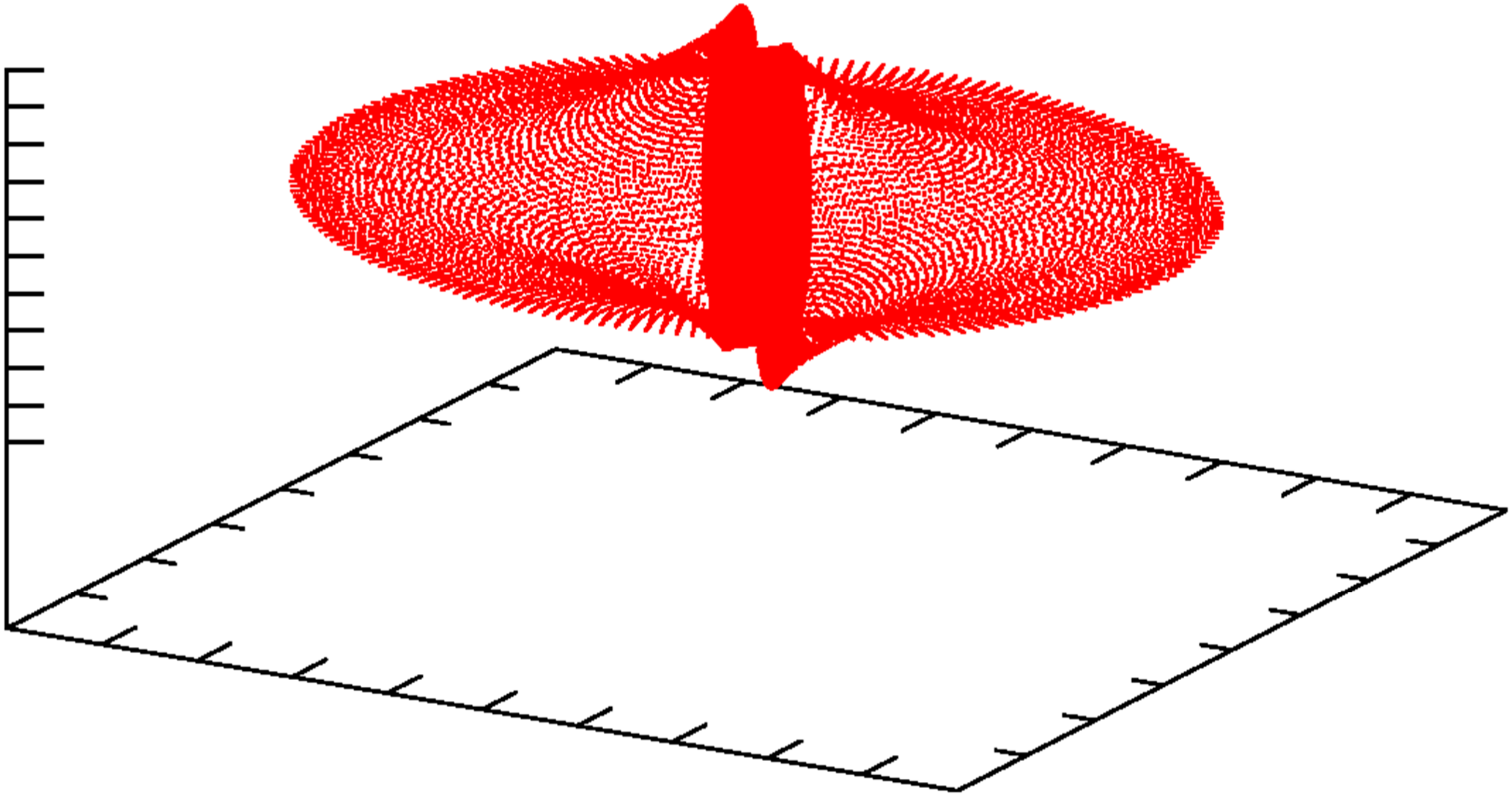


The caustic ring cross-section



D_{-4}

an elliptic umbilic catastrophe



On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be

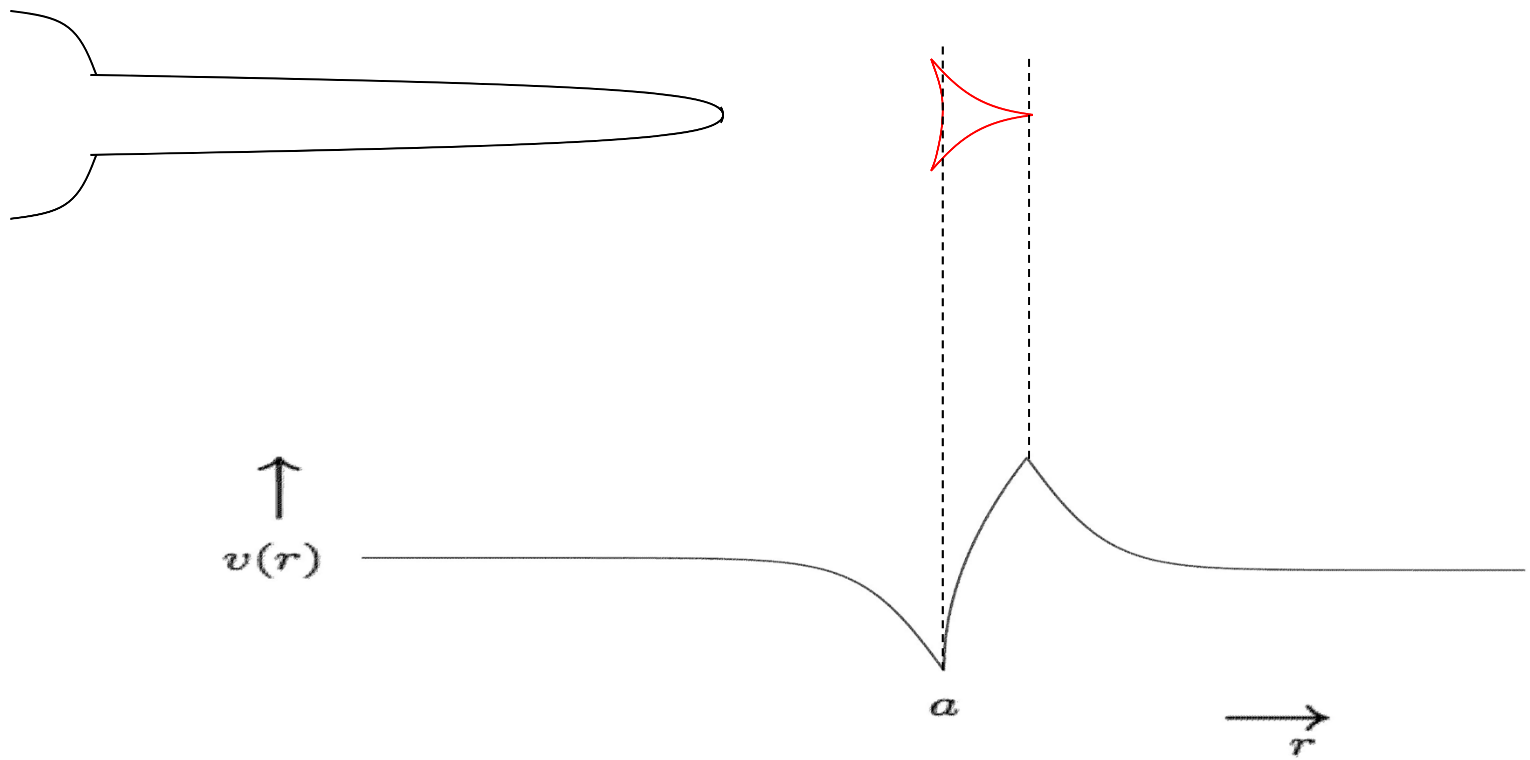
in the galactic plane

with radii $(n = 1, 2, 3\ldots)$

$$a_n = \frac{40\text{kpc}}{n}\left(\frac{v_{rot}}{220\text{km/s}}\right)\left(\frac{j_{max}}{0.18}\right)$$

$j_{max} \cong 0.18$ was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.

Effect of a caustic ring of dark matter upon the galactic rotation curve

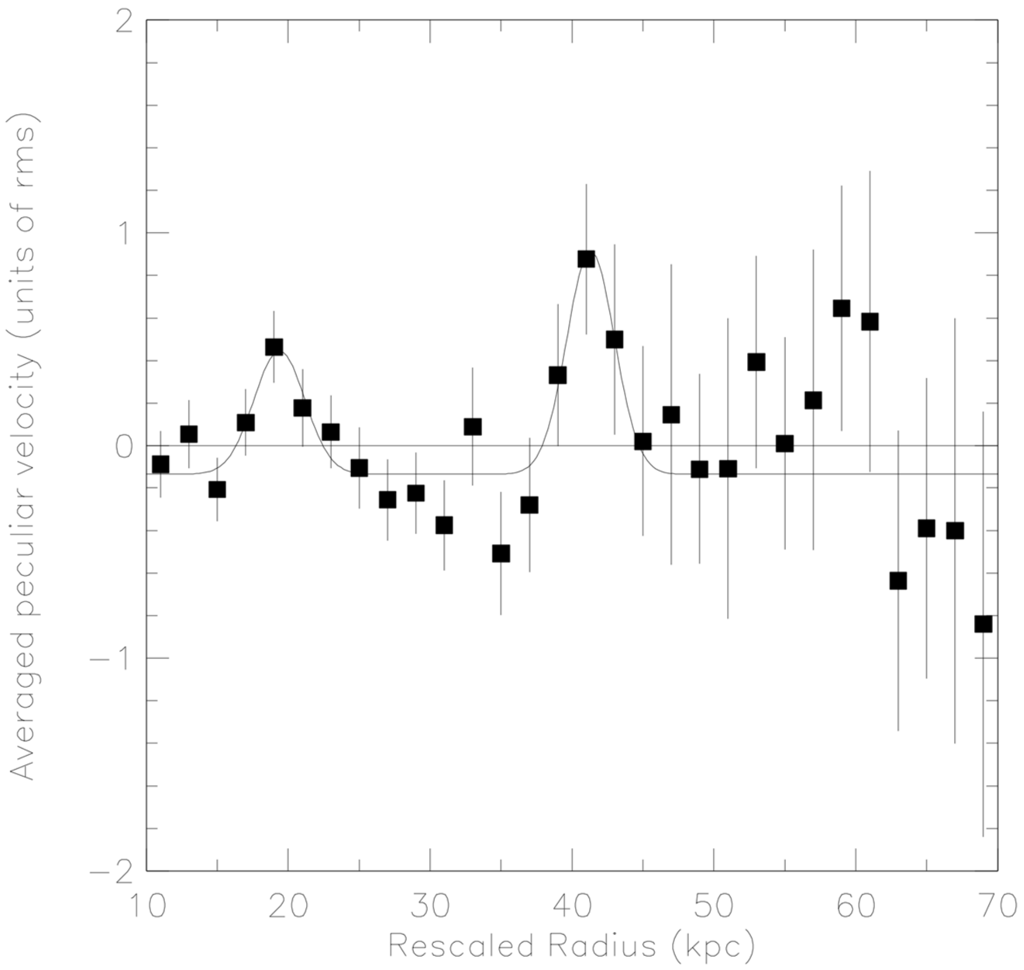


Composite rotation curve

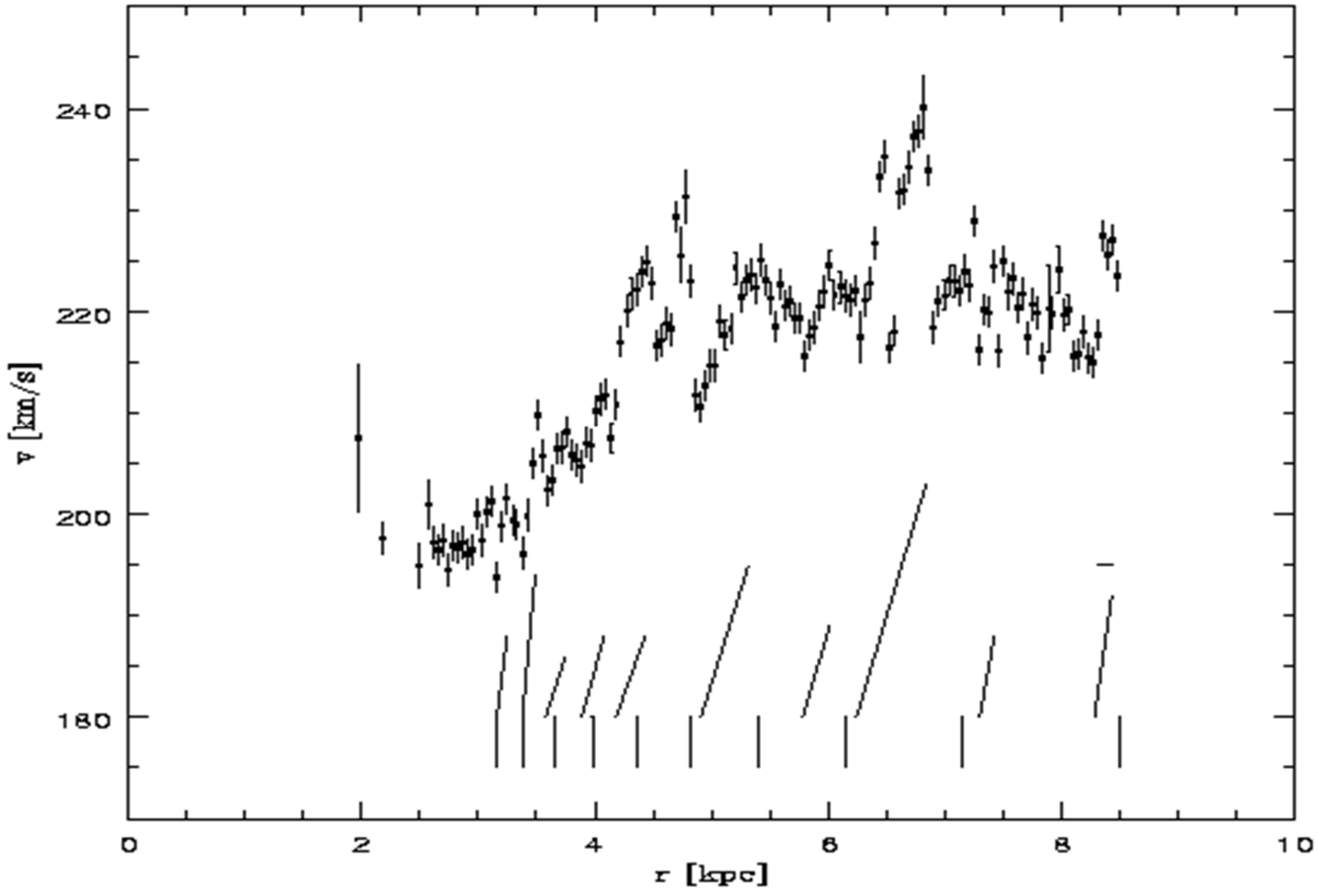
(W. Kinney and PS, astro-ph/9906049)

- combining data on 32 well measured extended external rotation curves

- scaled to our own galaxy



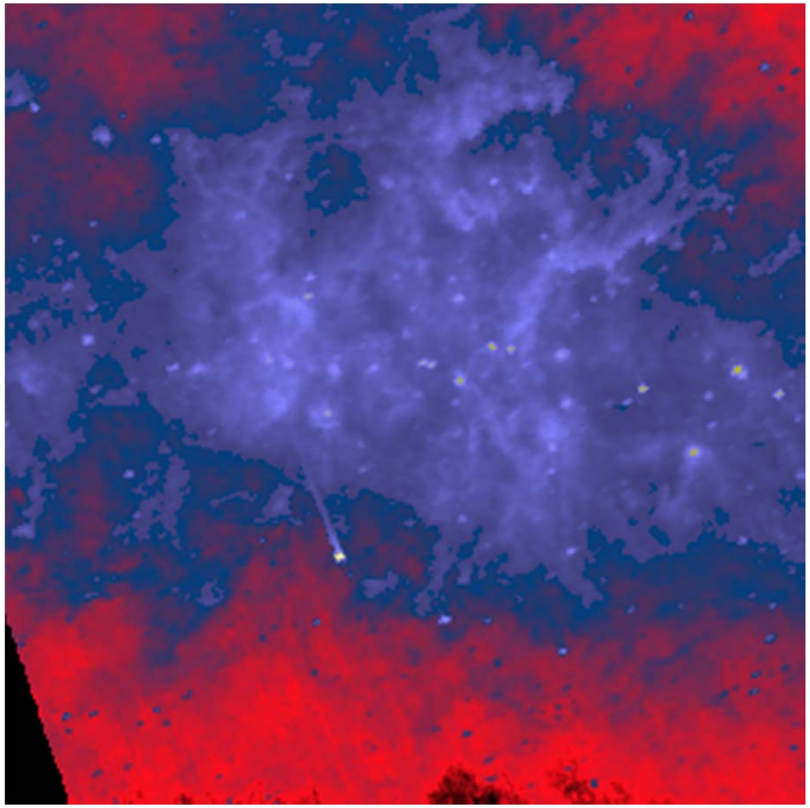
Inner Galactic rotation curve



from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)

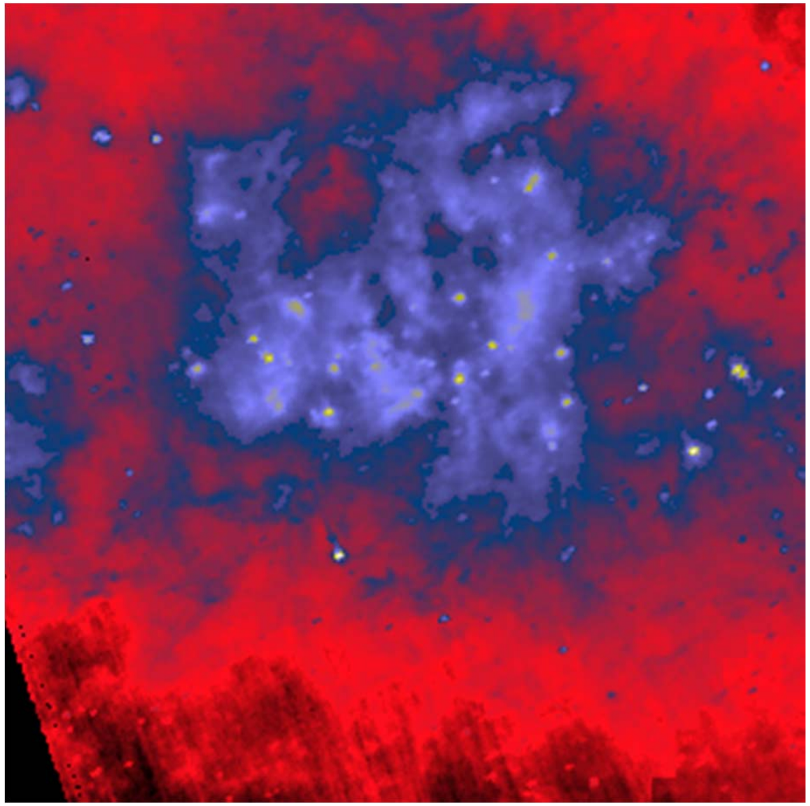
IRAS 12 μm

$(l, b) = (80^{o}, 0^{o})$ $10^{o} \times 10^{o}$

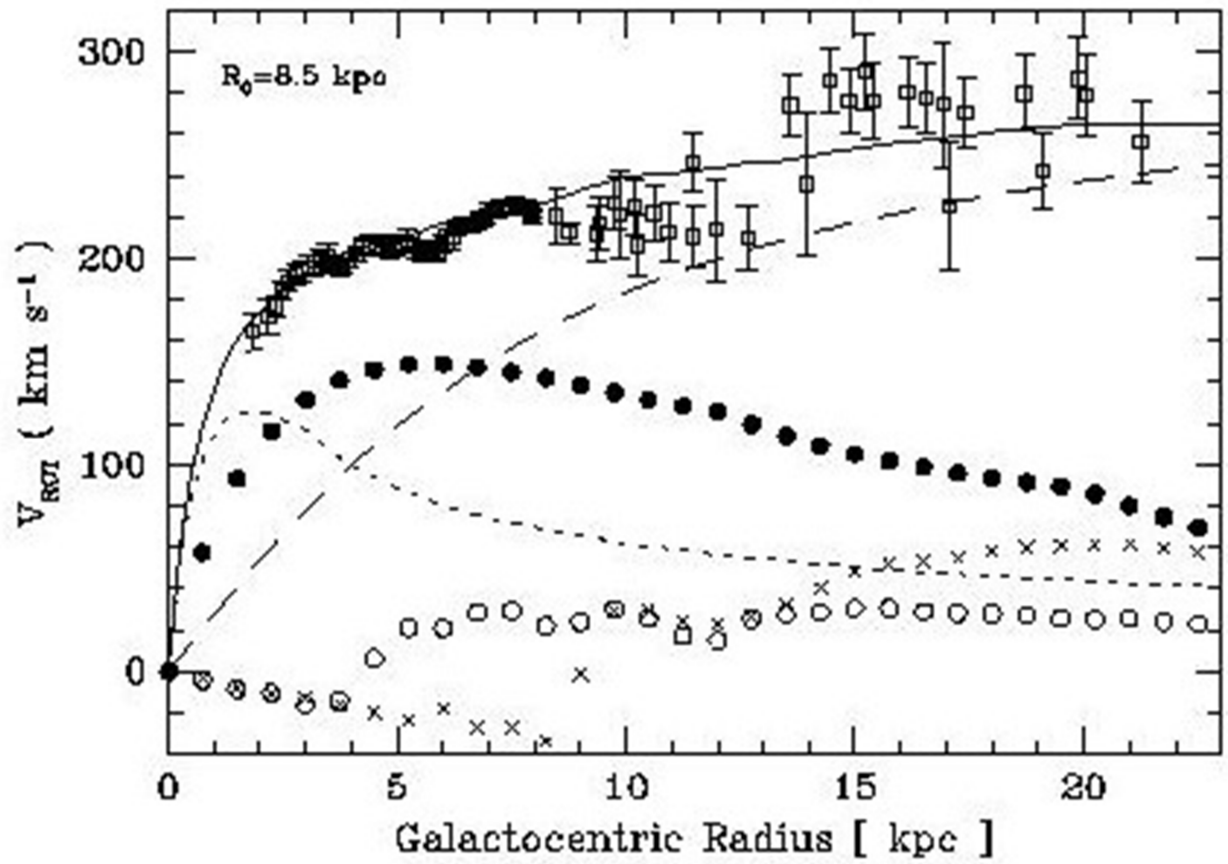


IRAS $25\ \mu$m

$(l, b) = (80^o, 0^o)$ $10^o \times 10^o$



Outer Galactic rotation curve



R.P. Olling and M.R. Merrifield, MNRAS 311 (2000) 361

Monoceros Ring of stars

H. Newberg et al. 2002; B. Yanny et al., 2003; R.A. Ibata et al., 2003;
H.J. Rocha-Pinto et al, 2003; J.D. Crane et al., 2003; N.F. Martin et al., 2005

in the Galactic plane

at galactocentric distance $r \simeq 20\,\mathrm{kpc}$

appears circular, actually seen for $100^0 < l < 270^0$

scale height of order 1 kpc

velocity dispersion of order 20 km/s

may be caused by the n = 2 caustic ring of dark matter (A. Natarajan and P.S. '07)

Rotation curve of Andromeda Galaxy

from L. Chemin, C. Carignan & T. Foster, arXiv: 0909.3846

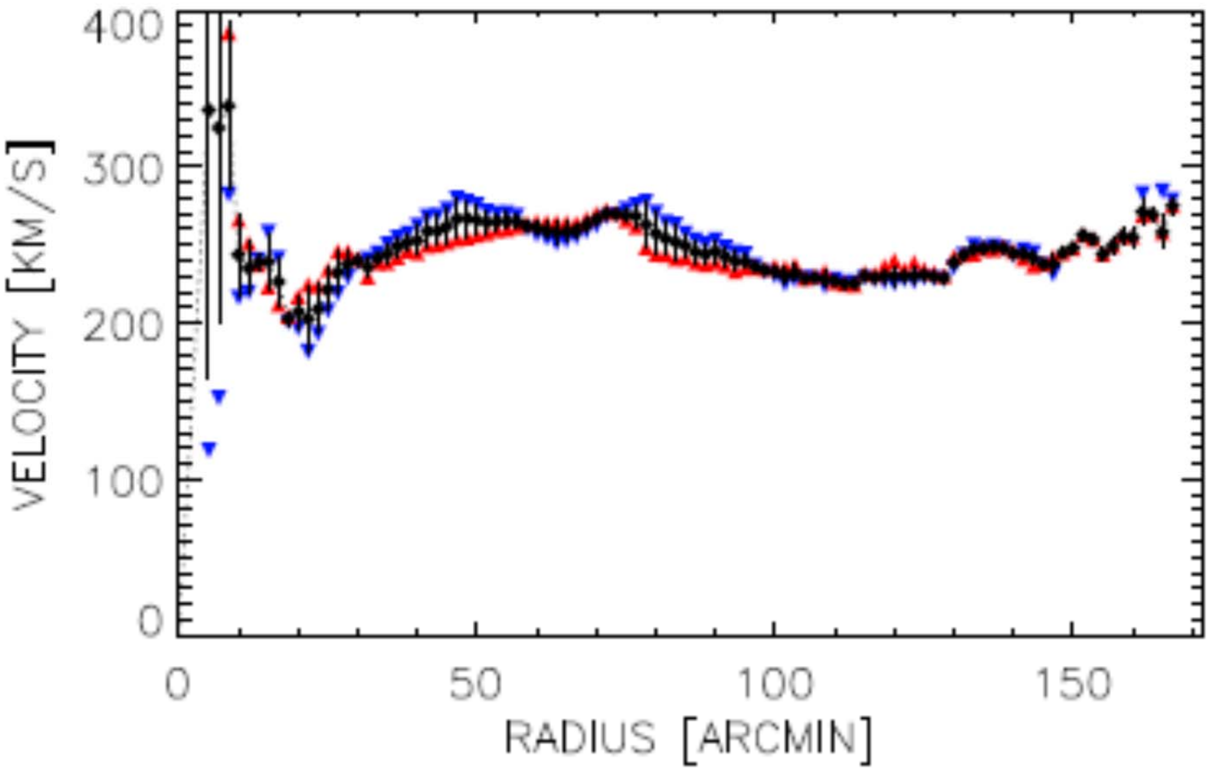
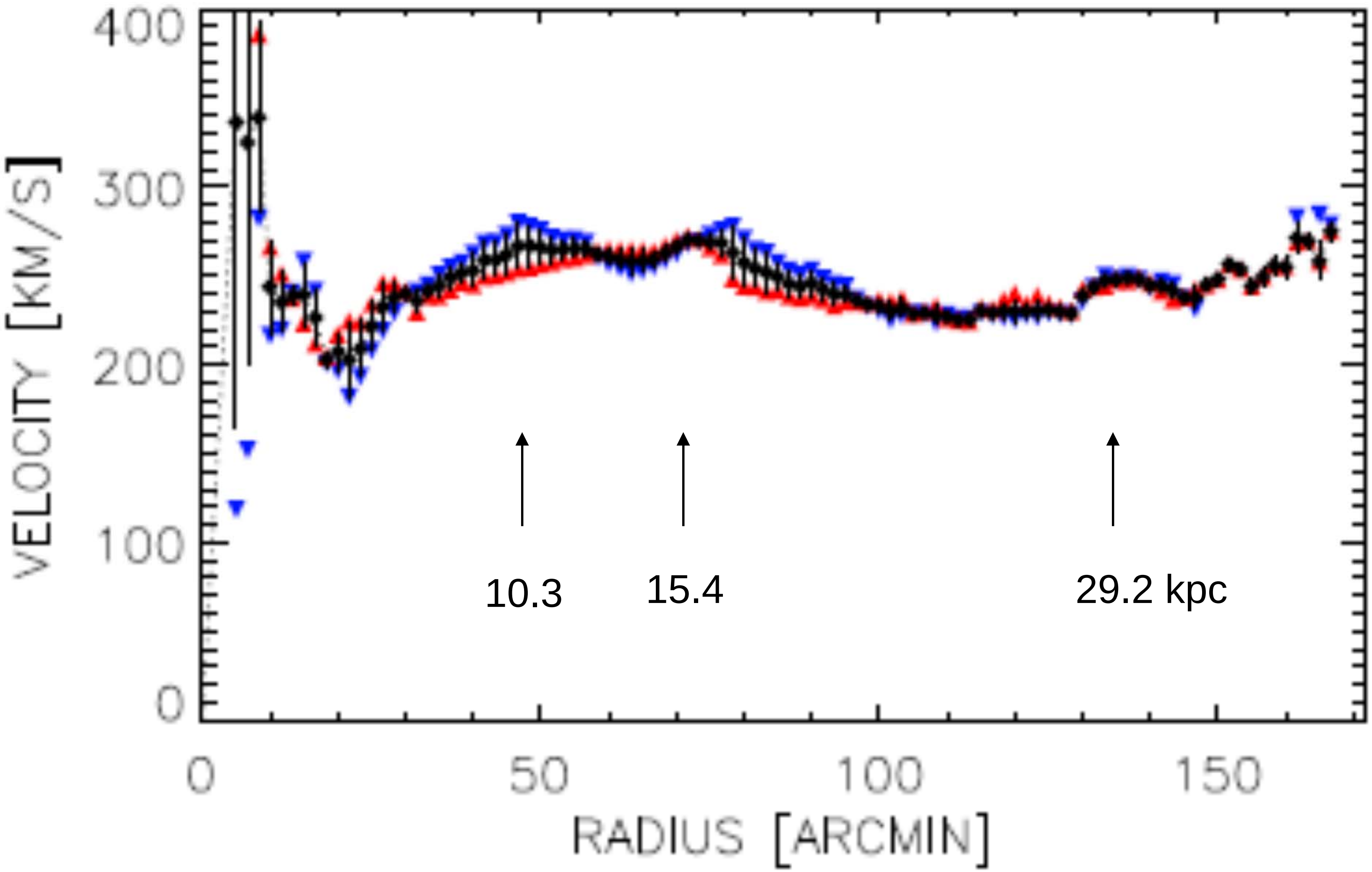


FIG. 10.— HI rotation curve of Messier 31. Filled diamonds are for both halves of the disc fitted simultaneously while blue downward/red upward triangles are for the approaching/receding sides fitted separately (respectively).



10 arcmin = 2.2 kpc

The caustic ring halo model assumes

- net overall rotation
- axial symmetry
- self-similarity

The specific angular momentum distribution on the turnaround sphere

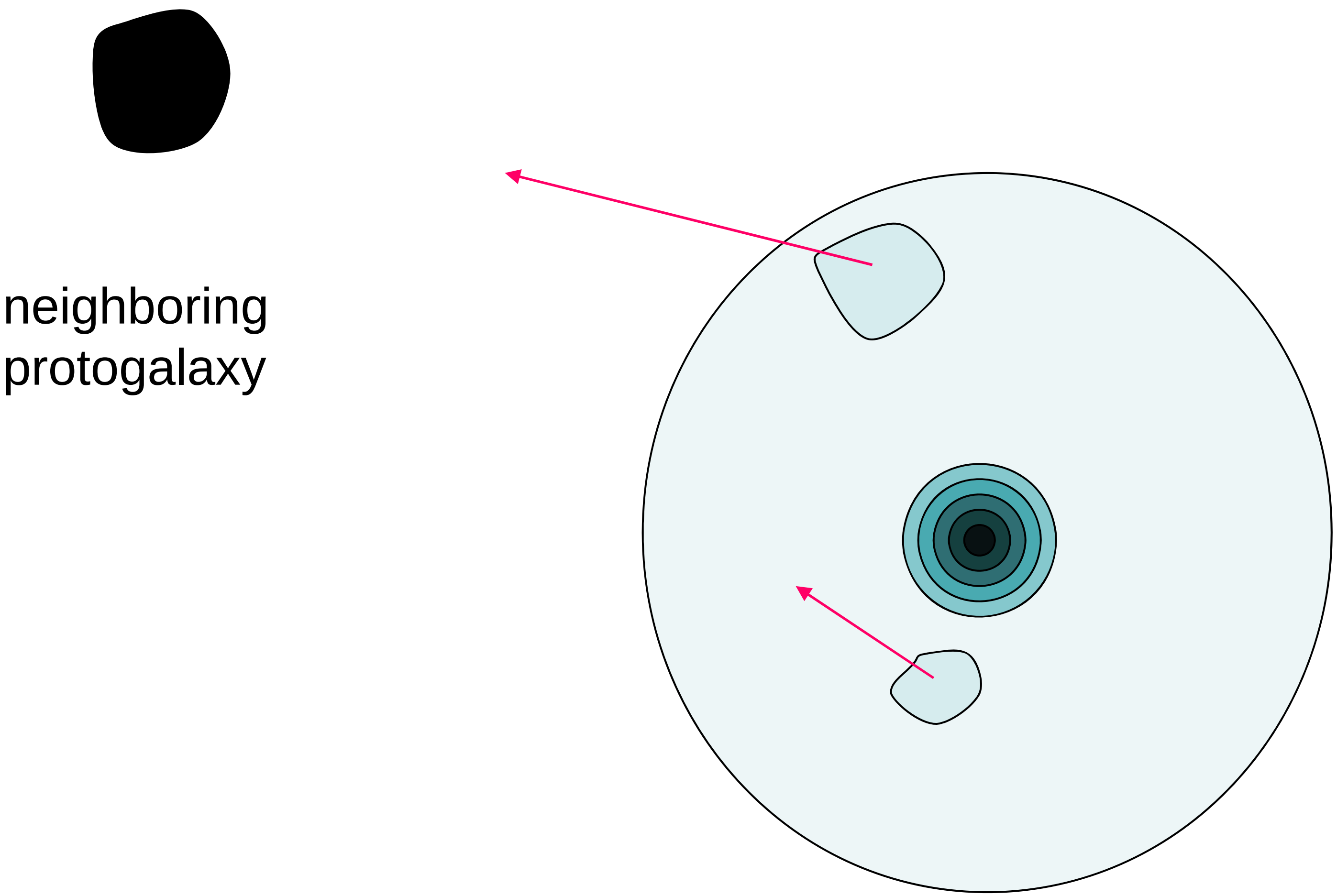
$$\vec{\ell}(\hat{n},t) = j_{\max} \; \hat{n}\times(\hat{z}\times\hat{n}) \; \frac{R(t)^2}{t}$$

$$R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$$

$$0.25 < \varepsilon < 0.35$$

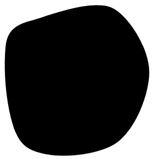
Is it plausible in the context of tidal torque theory?

Tidal torque theory

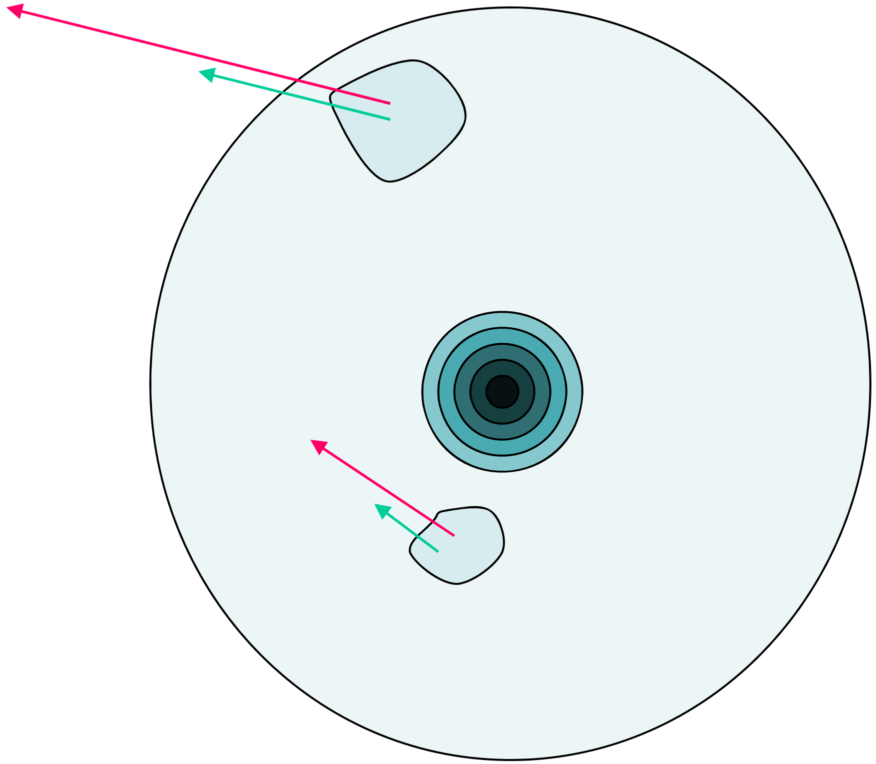


Stromberg 1934; Hoyle 1947; Peebles 1969, 1971

Tidal torque theory with ordinary CDM



neighboring protogalaxy



$$\vec{\nabla} \times \vec{v} = 0$$

the velocity field remains irrotational

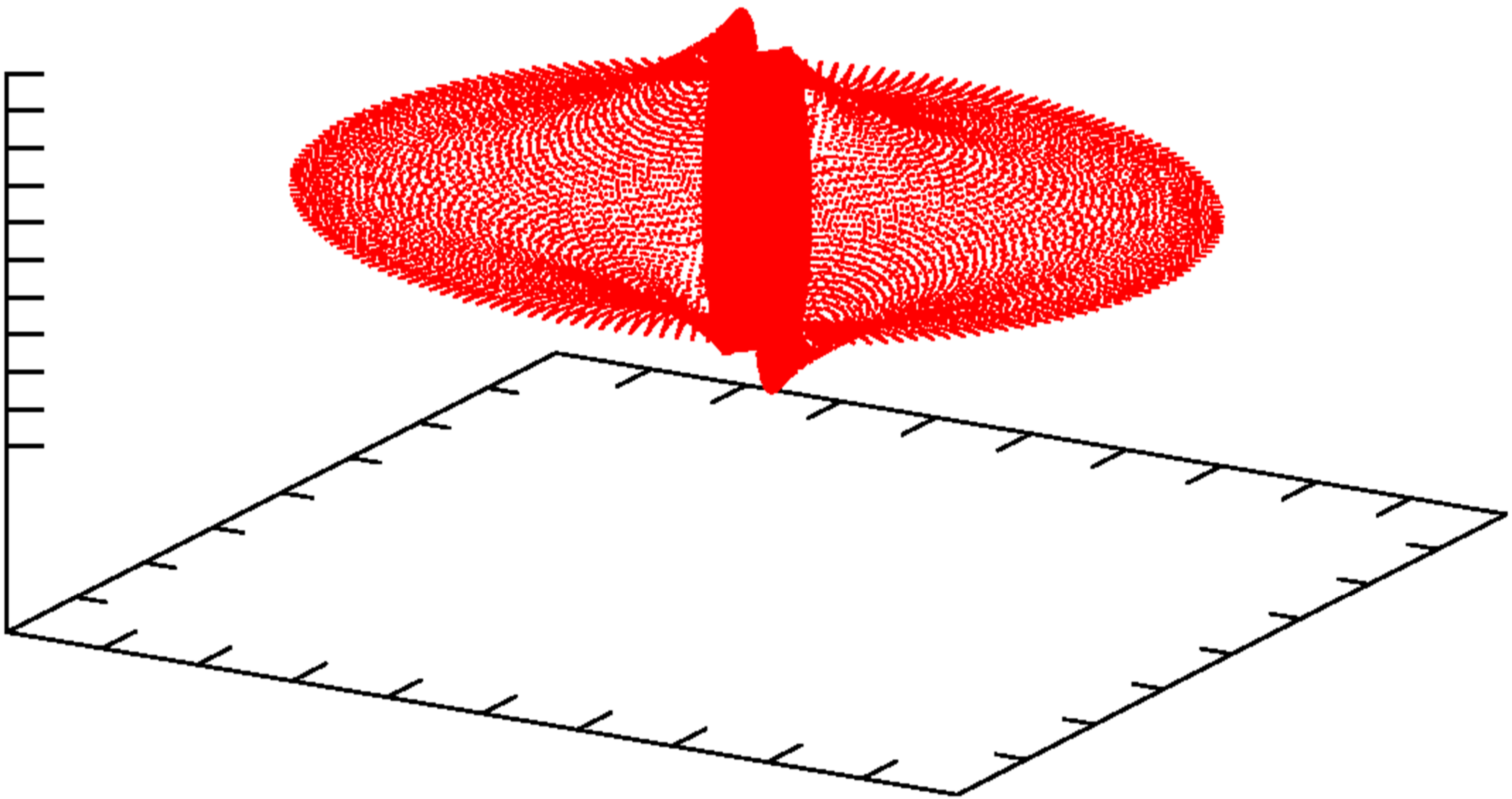
For collisionless particles

$$\frac{d\vec{\mathrm{v}}}{dt}(\vec{r},t) = \frac{\partial\vec{\mathrm{v}}}{\partial t}(\vec{r},t) + \left(\vec{\mathrm{v}}(\vec{r},t)\cdot\vec{\nabla}\right)\vec{\mathrm{v}}(\vec{r},t)$$

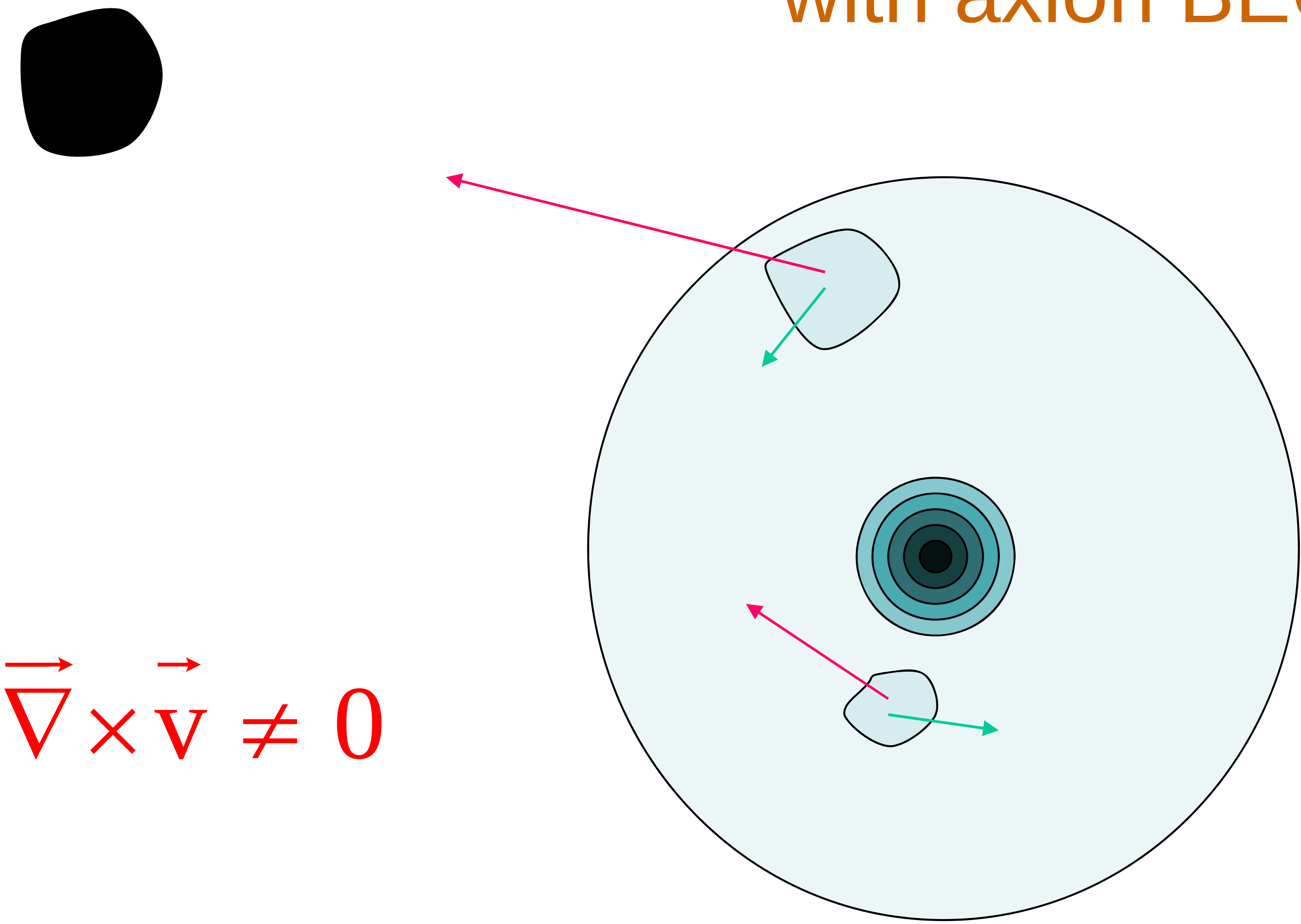
$$= -\vec{\nabla}\Phi(\vec{r},t)$$

If $\vec{\nabla}\times\vec{\mathrm{v}} = 0$ initially,

then $\vec{\nabla}\times\vec{\mathrm{v}} = 0$ for ever after.



Tidal torque theory
with axion BEC



$$\vec{\nabla} \times \vec{v} \neq 0$$

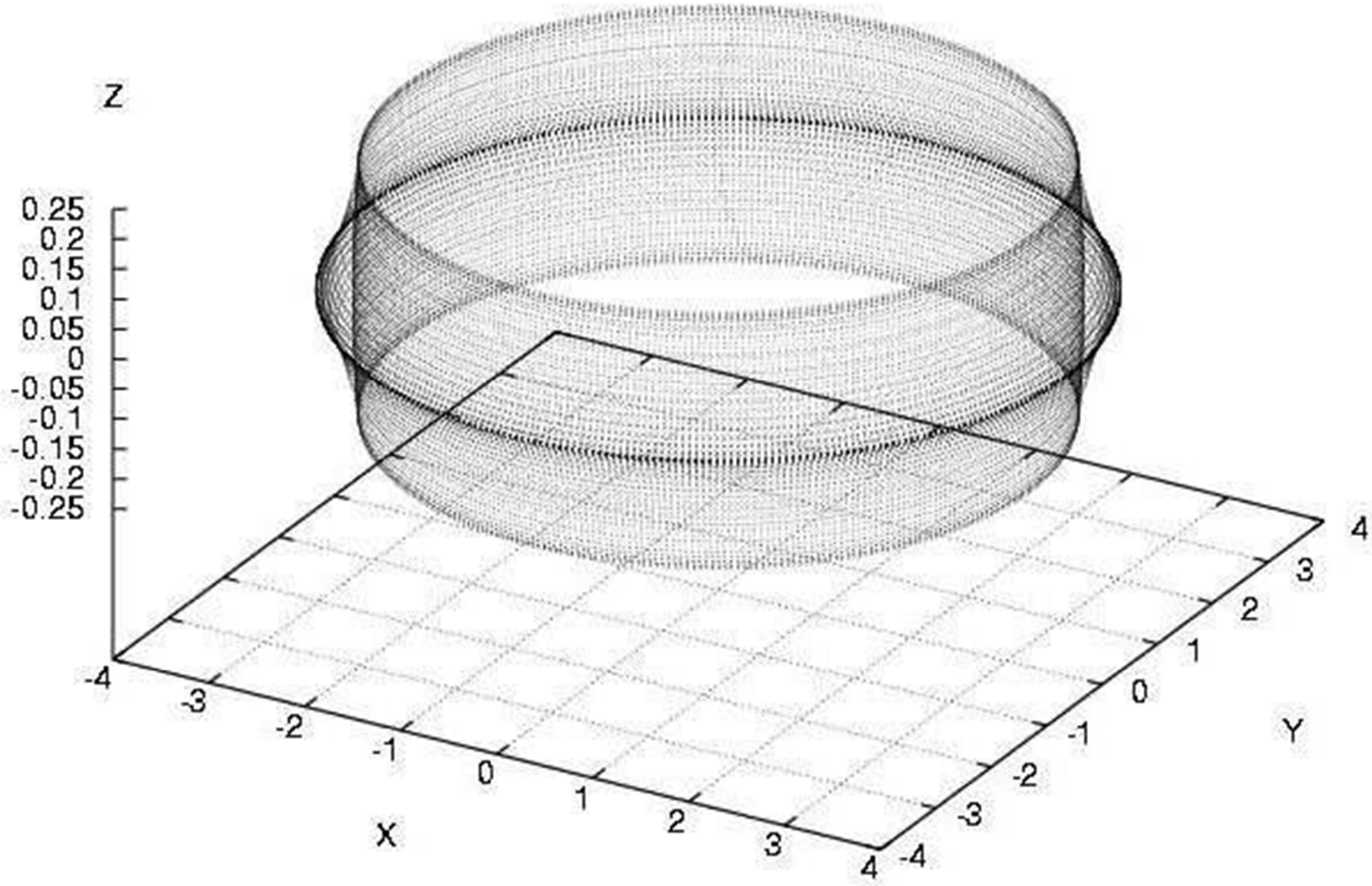
net overall rotation is obtained because, in the lowest energy state,
all axions fall with the same angular momentum

For axion BEC

$$E = \sum_{i=1}^{N} \frac{L_i^{\ 2}}{2I}$$

is minimized for given $L = \sum_{i=1}^{N} L_i$

when $L_1 = L_2 = L_3 = \ \dots \ = L_N$.


Z
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-4
-3
-2
-1
0
1
2
3
4
X
4
3
2
1
0
-1
-2
-3
-4
Y

The specific angular momentum distribution on the turnaround sphere

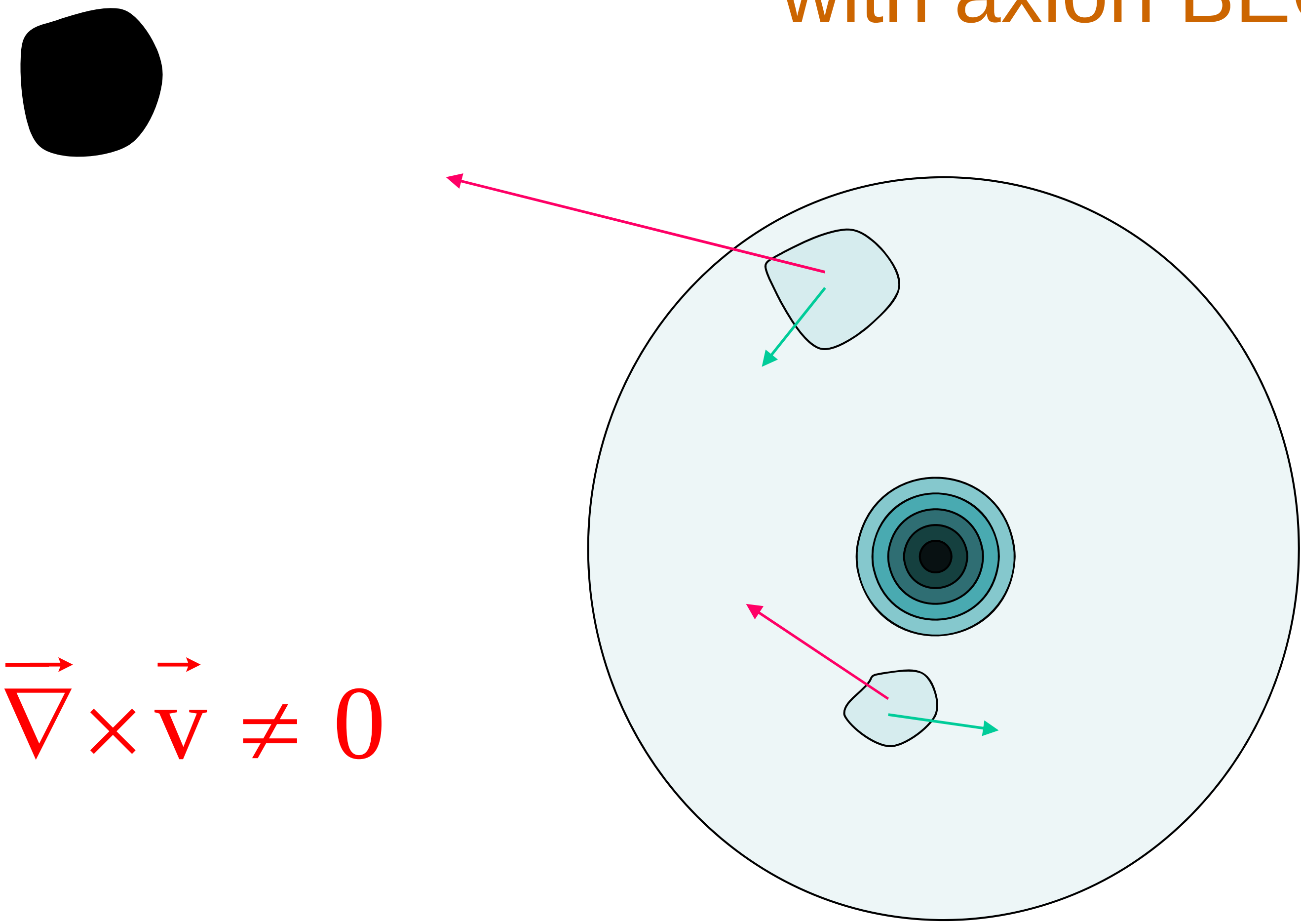
$$\vec{\ell}(\hat{n},t) = j_{\max} \ \hat{n}\times(\hat{z}\times\hat{n}) \ \frac{R(t)^2}{t}$$

$$R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$$

$$0.25 < \varepsilon < 0.35$$

Is it plausible in the context of tidal torque theory?

Tidal torque theory with axion BEC



$$\vec{\nabla} \times \vec{v} \neq 0$$

net overall rotation is obtained because, in the lowest energy state, all axions fall with the same angular momentum

Magnitude of angular momentum

$$\lambda = \frac{L\,|E|^{\frac{1}{2}}}{G\,M^{\frac{5}{2}}} = \sqrt{\frac{6}{5-3\varepsilon}}\;\frac{8}{10+3\varepsilon}\;\frac{1}{\pi}\;j_{\max}$$

$$\lambda \approx 0.05 \qquad\qquad j_{\max} \simeq 0.18$$

G. Efstathiou et al. 1979, 1987 — from caustic rings

fits perfectly ($0.25 < \varepsilon < 0.35$)

The specific angular momentum distribution on the turnaround sphere

$$\vec{\ell}(\hat{n},t) = j_{\max} \; \hat{n}\times(\hat{z}\times\hat{n}) \; \frac{R(t)^2}{t}$$

$$R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$$

$$0.25 < \varepsilon < 0.35$$

Is it plausible in the context of tidal torque theory?

Self-Similarity

$$\vec{\tau}(t) = \int_{V(t)} d^3 r \;\; \delta\rho(\vec{r},t) \;\; \vec{r} \times (-\vec{\nabla}\phi(\vec{r},t)$$

a comoving volume

$$\vec{r} = a(t)\,\vec{x} \qquad\qquad \phi(\vec{r} = a(t)\,\vec{x},\; t) = \phi(\vec{x})$$

$$\delta(\vec{r},t) \equiv \frac{\delta\rho(\vec{r},t)}{\rho_0(t)} \qquad\qquad \delta(\vec{r} = a(t)\,\vec{x},\; t) = a(t)\,\delta(\vec{x})$$

$$\vec{\tau}(t) = \rho_0(t)\,a(t)^4 \int_V d^3x \;\; \delta(\vec{x}) \;\; \vec{x} \times (-\vec{\nabla}_x\,\phi(\vec{x}))$$

Self-Similarity (yes!)

$$\vec{\tau}(t) \propto \hat{z}\, a(t) \propto \hat{z}\ t^{\frac{2}{3}}$$

$$\vec{L}(t) \propto \hat{z}\ t^{\frac{5}{3}}$$

time-independent axis of rotation

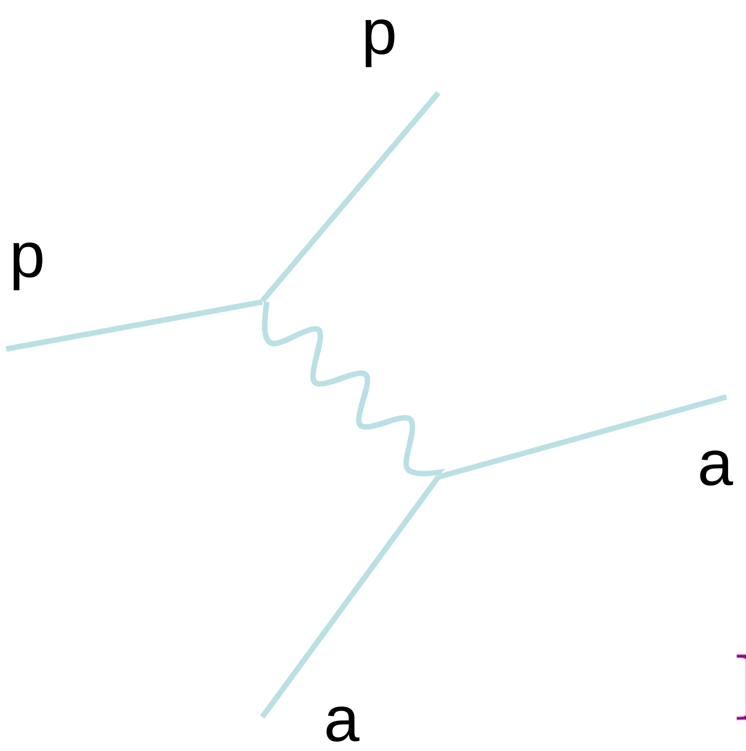
$$\vec{\ell}(\hat{n},t) \propto \frac{R(t)^2}{t} \propto t^{\frac{1}{3}+\frac{4}{9\varepsilon}} = t^{\frac{5}{3}}$$

provided $\varepsilon = 0.33$

Conclusion:

The dark matter looks like axions

Baryons and photons enter into thermal contact with the axion BEC



$$\Gamma \sim 4\pi G \, n \, m \, \omega \, \frac{1}{\Delta p}$$

O. Erken, PS, H.Tam and Q. Yang - arXiv:1104.4507

Photons, baryons and axions all reach the same temperature before decoupling

photons cool

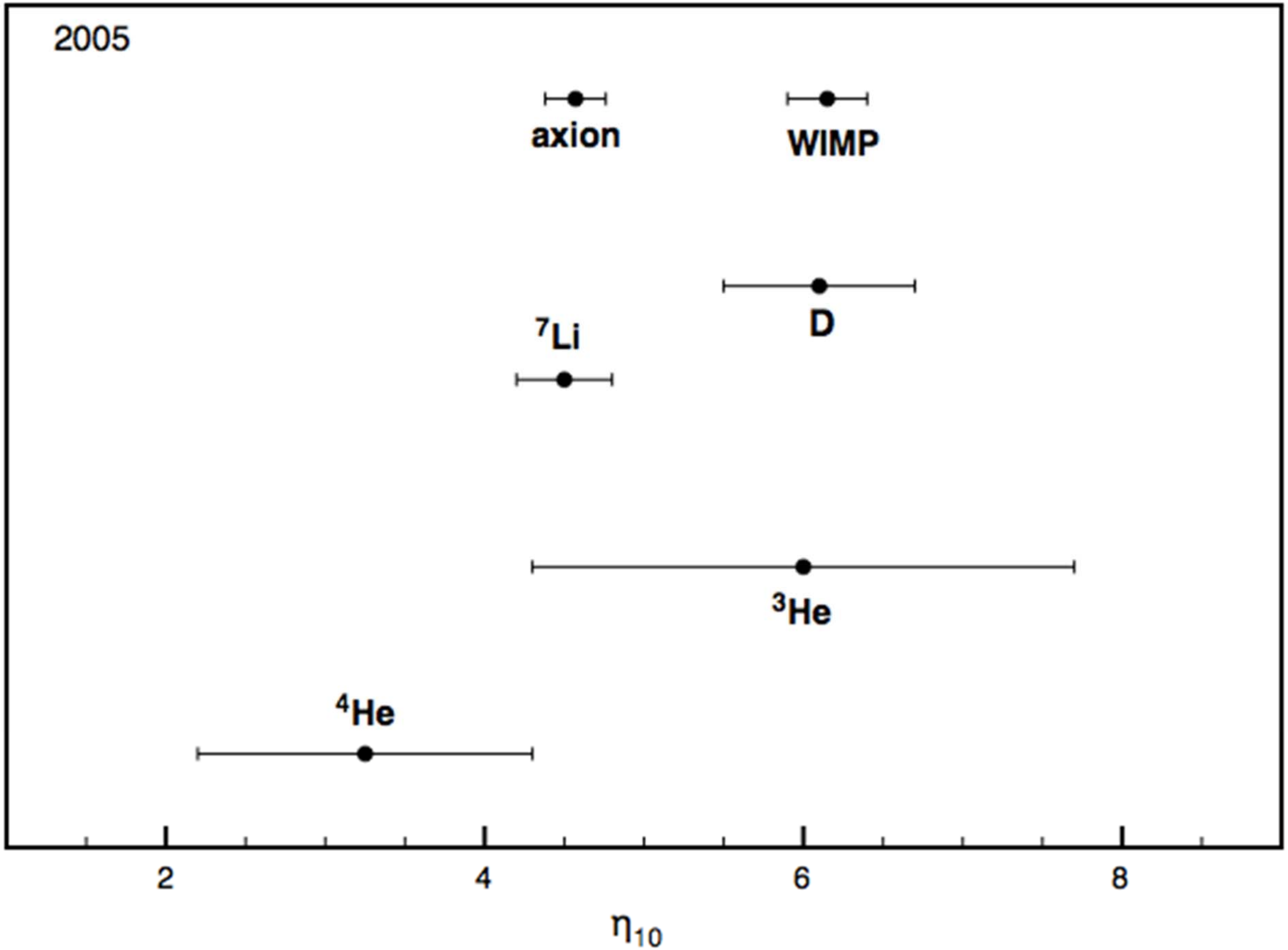
$$T_{\gamma,\mathrm{f}} \;=\; 0.904\, T_{\gamma,\mathrm{i}}$$

baryon to photon ratio

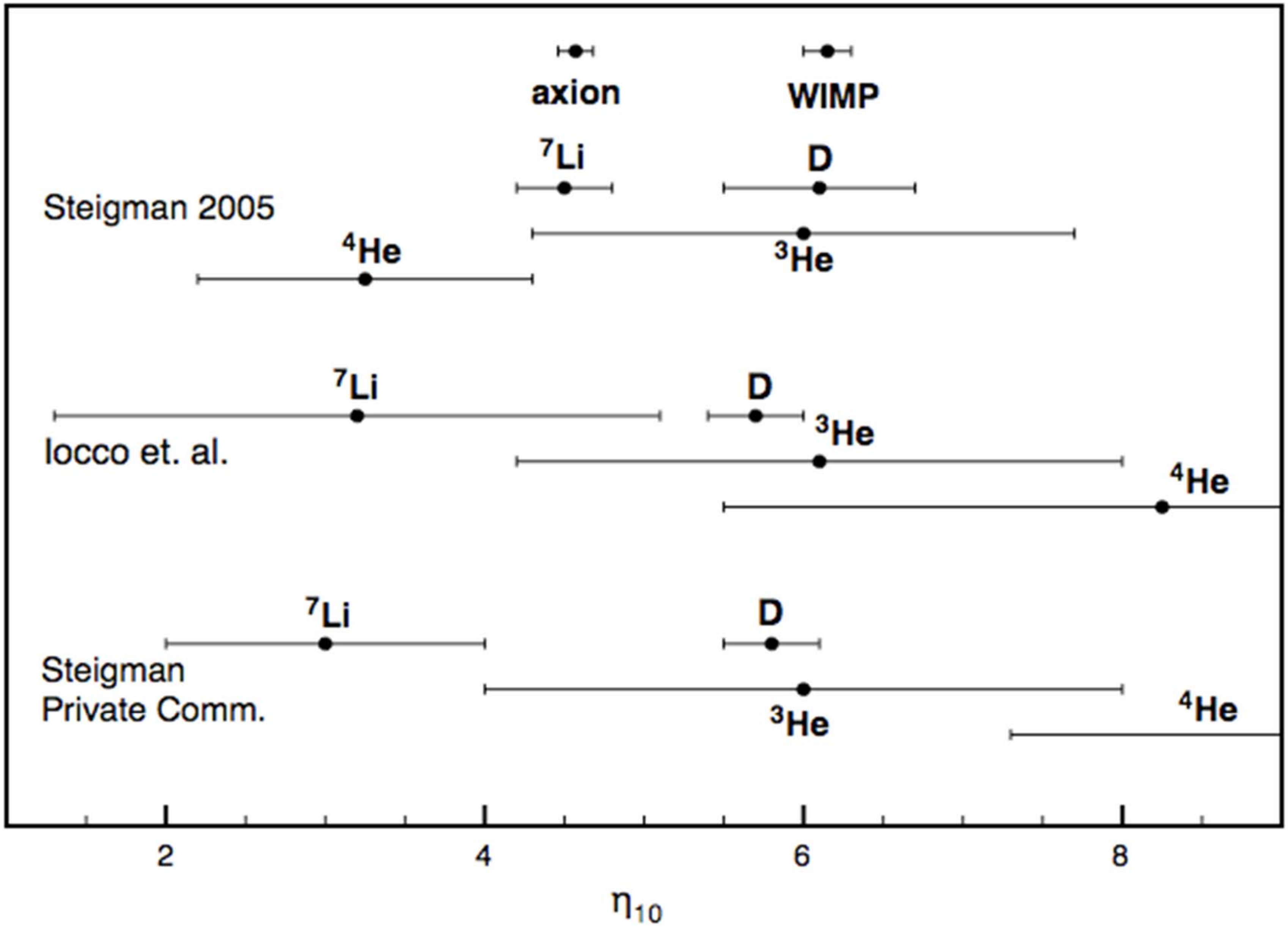
$$\eta_{\mathrm{BBN}} \;=\; 0.738\; \eta_{\mathrm{WMAP}}$$

effective number of neutrinos

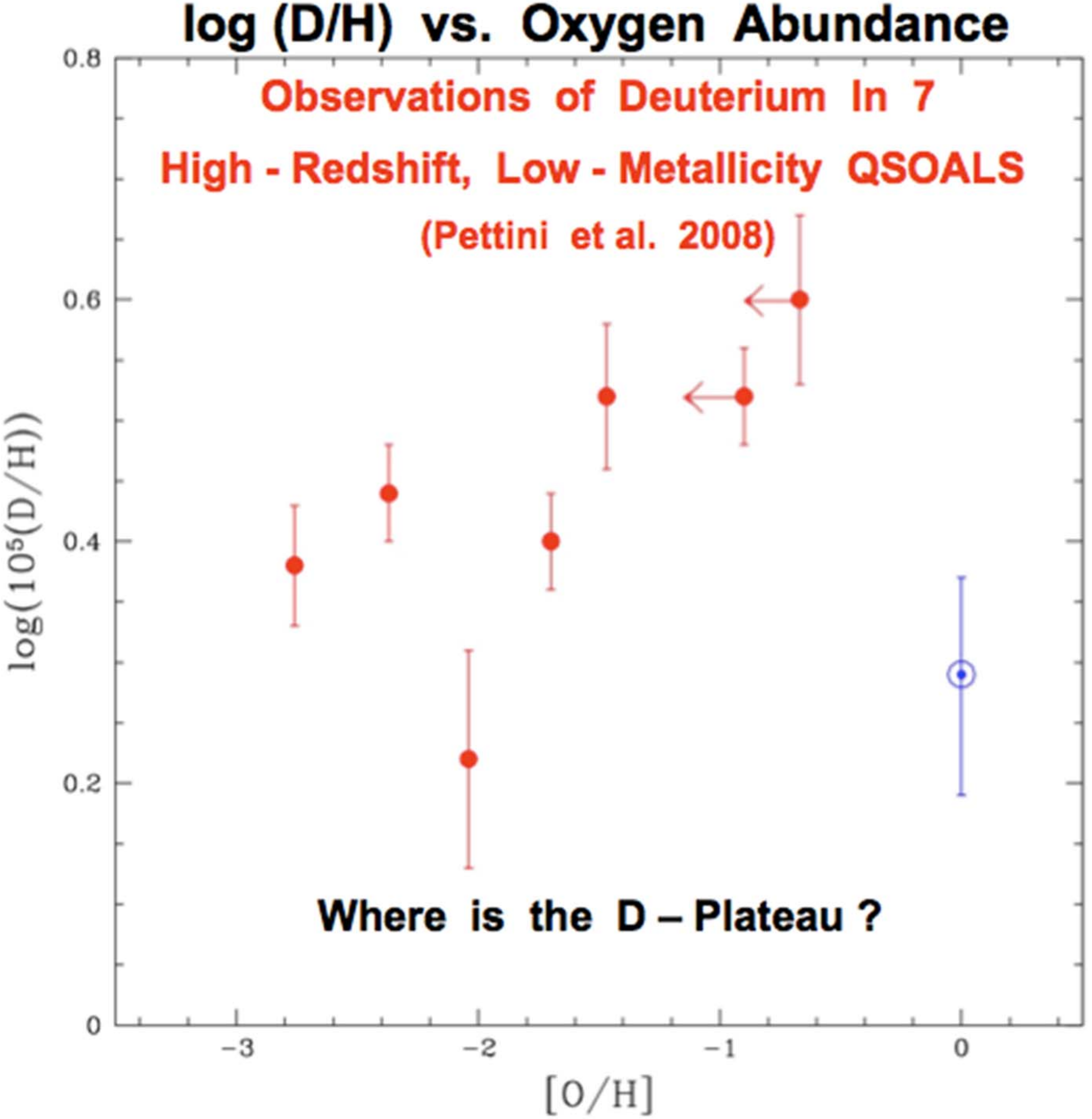
$$N_{\nu,\mathrm{eff}} \;=\; 6.7$$



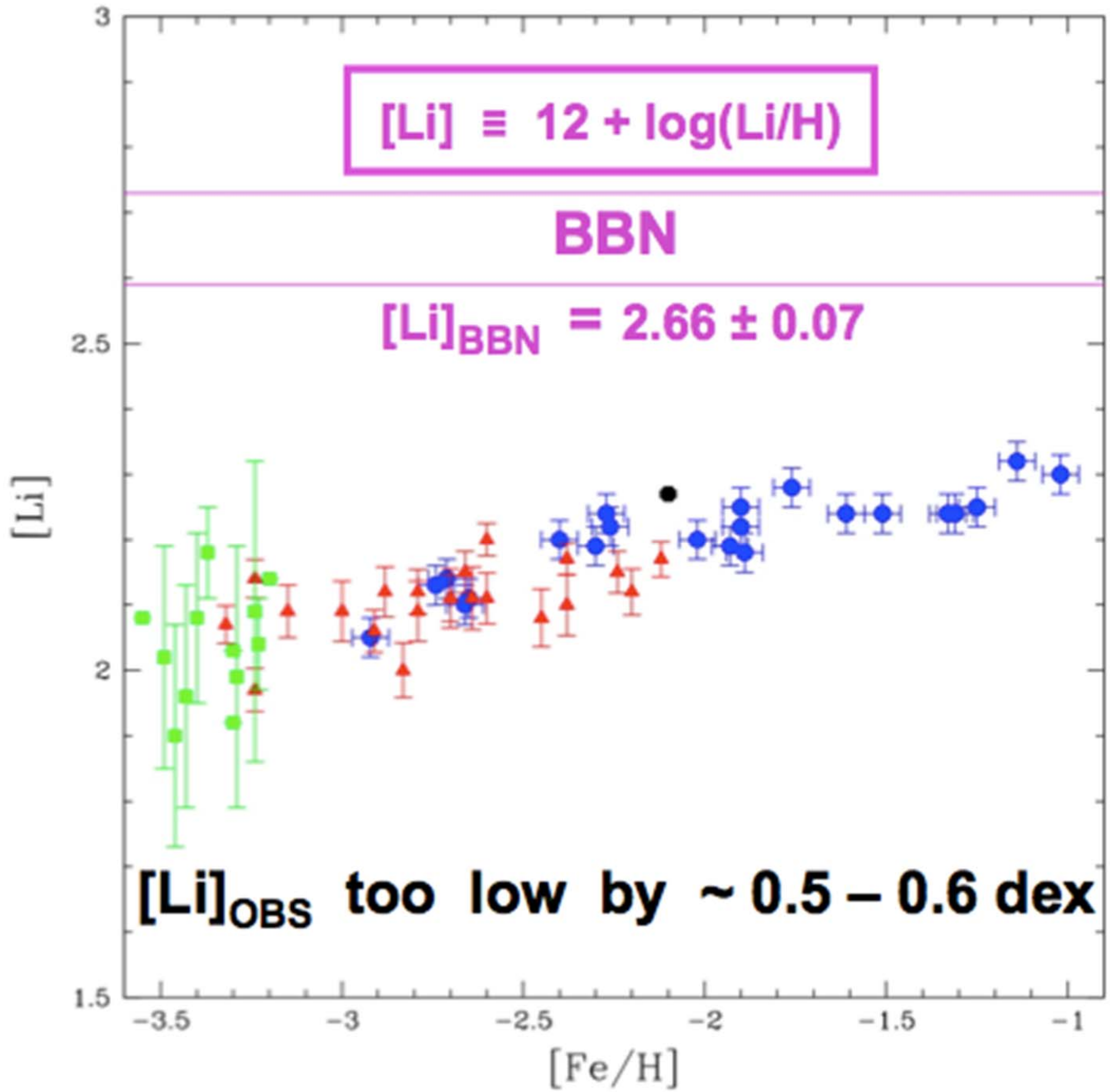
2005
axion
WIMP
D
^{7}Li
^{3}He
^{4}He
2
4
6
8
η_{10}



axion
WIMP
Steigman 2005
^{7}Li
D
^{4}He
^{3}He
Iocco et. al.
^{7}Li
D
^{3}He
^{4}He
Steigman
Private Comm.
^{7}Li
D
^{3}He
^{4}He
2
4
6
8
η_{10}



log (D/H) vs. Oxygen Abundance
Observations of Deuterium In 7
High - Redshift, Low - Metallicity QSOALS
(Pettini et al. 2008)
Where is the D – Plateau ?
log(10^5(D/H))
[O/H]
0.8
0.6
0.4
0.2
0
−3
−2
−1
0



[Li] ≡ 12 + log(Li/H)
BBN
$[\mathrm{Li}]_{\mathrm{BBN}} = 2.66 \pm 0.07$
$[\mathrm{Li}]_{\mathrm{OBS}}$ too low by ~ 0.5 – 0.6 dex
[Li]
[Fe/H]
3
2.5
2
1.5
−3.5
−3
−2.5
−2
−1.5
−1

Effective number of neutrinos

$$\rho_{\rm rad} = \rho_\gamma + \rho_a + \rho_\nu$$
$$= \rho_\gamma \left[1 + N_{\rm eff} \frac{7}{8} \left(\frac{4}{11}\right)^{\frac{4}{3}}\right]$$

$$N_{\rm eff} = 6.7$$

WMAP 7 year: $4.34 \pm 0.87 \ \ (68\% \ \mathrm{CL})$

J. Hamann et al. (SDSS): $4.8 \pm 2.0 \ \ (95\% \ \mathrm{CL})$

Atacama Cosmology Telescope: $5.3 \pm 1.3 \ (68\% \ \mathrm{CL})$

we will see …