The photon polarization tensor in external fields

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Introduction to $\Pi^{\mu\nu}$

$\Pi^{\mu\nu}$ in an external magnetic field - the general situation

$\rightarrow$ we aim at insights "beyond the light-cone"

$\rightarrow$ necessary to tackle problems formulated in position space (e.g., boundary conditions in position space: walls, etc.)

$\Pi^{\mu\nu}$ in an external magnetic field - a specific alignment

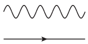
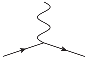
$\leftrightarrow$ a starting point where we can obtain some insights

Conclusions & Outlook

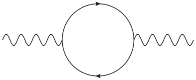
Introduction to $\Pi^{\mu\nu}$

The photon polarization tensor

consider quantum electrodynamics (QED)

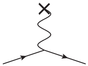
propagators :  vertex : 

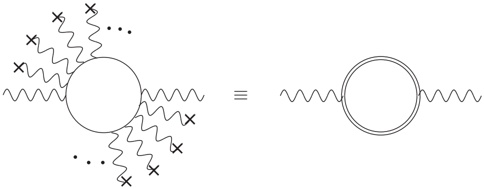
- 1-loop polarization tensor (in the absence of external fields)

$$\Pi^{\mu\nu}(k) =$$ 

- in the presence of an external field

  $\rightarrow$ 1-loop polarization tensor

external field : 



Photon propagation in the quantum vacuum

$\Pi^{\mu\nu}$ is the central input to an effective theory for photon propagation in the quantum vacuum

$$\mathcal{L}_{\text{eff}}[A] = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \int_{x'} A_\mu(x) \, \underset{\substack{\uparrow \\ \text{vacuum fluctuations}}}{\Pi^{\mu\nu}(x,x')} \, A_\nu(x')$$

(here A_μ denotes a classical, macroscopic field)

- gives rise to modified speeds of light in external fields
- accounts for pair creation effects $\sim$ imaginary part

without external fields: $\Pi^{\mu\nu}$ easily evaluated in momentum space
$\leftrightarrow$ in the presence of (constant) external fields: rather involved

$\Pi^{\mu\nu}$ in an external magnetic field - the general situation

The photon polarization tensor for B=const.

$\Pi^{\mu\nu}$ for pure $B = |\mathbf{B}|$=const. is known explicitly, but

- in general cannot be tackled analytically
- parameter integral $\sim$ momentum distribution within loop "ν" and proper-time integral (or infinite series over Landau levels)

  [J. S. Schwinger; Phys. Rev. **82**, 664 (1951)]

- we focus on the proper-time "s" representation

  [L. F. Urrutia; Phys. Rev. D **17**, 1977 (1978)]

$$\Pi^{\mu\nu}(k) \sim P^{\mu\nu}(k) \int_0^\infty \frac{\mathrm{d}s}{s} \int_{-1}^{+1} \frac{\mathrm{d}\nu}{2} \, \mathrm{e}^{-\mathrm{i}\Phi_0(z,\nu;k)s} \, f(z,\nu)$$

where $z \equiv eBs$, and "phase factor" $\quad [\, k^2 = \mathbf{k}^2 - \omega^2 \,]$

$$\Phi_0(z,\nu;k) = m^2 + \frac{1-\nu^2}{4}\left(\mathbf{k}_\parallel^2 - \omega^2\right) + \frac{\cos\nu z - \cos z}{2z\sin z}\,\mathbf{k}_\perp^2$$

Structure of $\Pi^{\mu\nu}$ for B=const.

in general $\Pi^{\mu\nu}$ has the following structure:

[W. Dittrich and H. Gies; Springer Tracts Mod. Phys. **166**, 1 (2000)

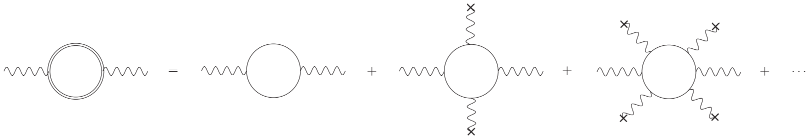
$$\Pi^{\mu\nu}(k) \;=\; \Pi_0(k)\,P_0^{\mu\nu} + \Pi_{\|}(k)\,P_{\|}^{\mu\nu} + \Pi_{\perp}(k)\,P_{\perp}^{\mu\nu}$$

- tensor structure in projectors $P_0^{\mu\nu}$, $P_{\|}^{\mu\nu}$ and $P_{\perp}^{\mu\nu}$ (depend on ω, $\mathbf{k}$ and $\mathbf{B}$)
- projectors span the transversal subspace
- scalar "components" $\Pi_0(k)$, $\Pi_{\|}(k)$ and $\Pi_{\perp}(k)$
- equations of motion for photons in 0, $\|$ and $\perp$ modes of simple form, e.g., $\Big(k^2 + \Pi_{\|}(k)\Big)\,A_{\|} = 0\,,$ etc.
- three independent polarization modes (cf. medium)

Approximations to $\Pi^{\mu\nu}$ for B=const.

several well-established approximations to $\Pi^{\mu\nu}$:

- perturbative expansion in # of field insertions



- Tsai & Erber ("small z expansion")

  [W. y. Tsai & T. Erber; Phys. Rev. D **10**, 492 (1974) & Phys. Rev. D **12**, 1132 (1975)]

  assumptions: "on-the-light-cone", i.e., $k^2 = 0 \quad \leftrightarrow \quad |\mathbf{k}| = \omega\,,$

  and originally $\frac{eB}{m^2} \ll 1 \quad \& \quad \frac{\omega^2 \sin^2\theta}{m^2} \gg 1\,,$

  but also insights for $\frac{\omega^2 \sin^2\theta}{eB} \gg 1$ alone

- "large z expansion"

  [A. E. Shabad; Annals Phys. **90**, 166 (1975) & arXiv:hep-th/0307214]

  $\rightarrow$ "projects out" lowest Landau level

$\Pi^{\mu\nu}$ in an external magnetic field

- a specific alignment

$\Pi^{\mu\nu}$ for B=const. and $\mathbf{k} \parallel \mathbf{B}$

recall the explicit structure of $\Pi^{\mu\nu}$, $[\, z = eBs \,]$

$$\Pi^{\mu\nu}(k) \;\sim\; P^{\mu\nu}(k) \int_0^\infty \frac{\mathrm{d}s}{s} \int_{-1}^{+1} \frac{\mathrm{d}\nu}{2}\, \mathrm{e}^{-\mathrm{i}\Phi_0(z,\nu;k)s}\, f(z,\nu)$$

where

$$\Phi_0(z,\nu;k) = m^2 + \frac{1-\nu^2}{4}\left(\mathbf{k}_\parallel^2 - \omega^2\right) + \underbrace{\frac{\cos\nu z - \cos z}{2z\sin z}\,\mathbf{k}_\perp^2}_{\substack{\uparrow \\ \text{vanishes for } \mathbf{k} \parallel \mathbf{B}}}$$

$\left(\text{side remark: } \xrightarrow{\text{“large z”}} \sim \frac{\mathbf{k}_\perp^2}{2eBis}\right)$

- $\Pi^{\mu\nu}$ simplifies significantly for $\mathbf{k} \parallel \mathbf{B}$
- full momentum dependence retained $(\omega, \mathbf{k})$

$\Pi^{\mu\nu}$ for B=const. and $\mathbf{k} \parallel \mathbf{B}$

here $\Pi^{\mu\nu}$ has the following structure:

$$\Pi^{\mu\nu}(k) \,=\, \Pi_{\llcorner}(k)\, P_{\llcorner}^{\mu\nu} + \Pi_{\pm}(k)\, \underbrace{\left(P_{+}^{\mu\nu} + P_{-}^{\mu\nu}\right)}_{\text{circular polarization } (\pm)}$$

the proper-time integration can be performed explicitly in this limit

- by means of a rotation into the Euclidean $s \to -\mathrm{i}s$

  [R. A. Cover & G. Kalman; Phys. Rev. Lett. **33**, 1113 (1974)]

  [W. y. Tsai & T. Erber; Act. Phys. Austr. **45**, 245 (1976)]

- and employing the "electromagnetic duality" $B \leftrightarrow \mathrm{i}E$

the results can be expressed through Digamma function $\Psi(\chi)$

- exact series representation $\leftrightarrow$ Landau-level structure
- zeroth Landau-level dominates large B behavior (unscreened for $\llcorner$ mode)

Conclusions & Outlook

Conclusions & Outlook

$\Pi^{\mu\nu}$ is a central quantity, certainly worthwhile to study

- modification of dispersion law
- accounts for pair creation effects

we aim at non-perturbative insights, retaining full k-dependence

- necessary whenever we want to transform into position space
- essential for LSW with virtual minicharges in external B-field ($\rightarrow$ talk by Babette Döbrich on Monday)
- so far rather unexplored $\dashrightarrow$ novel insights

it would be nice

- special alignment $\dashrightarrow$ general situation
- to go beyond B=const.
- ...

Thank you for your attention!