Photon propagation in a cold axion background (with and without a magnetic field)

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Outline

1.- Introdution

2.- Chern-Simons electrodynamics & Lorentz breaking

3.- Cold axions trigger Lorentz symmetry breaking

4.- Axion-induced Bremsstrahlung

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Introduction

Cold relic axions resulting from vacuum misalignment in the early universe is a popular and so far viable candidate to dark matter (e.g. Sikivie's talk).

Provided that the reheating temperature after inflation is below the Peccei-Quinn transition scale, in later times the axion evolves as

$$a(t) = a_0 \cos m_a t, \qquad \mathbf{k} = 0$$

$$\rho \simeq a_0^2 m_a^2$$

$$\rho \simeq 10^{-30}\mathrm{gcm}^{-3} \simeq 10^{-10}\mathrm{eV}^4, \qquad \rho^* \simeq 10^{-24}\mathrm{gcm}^{-3} \simeq 10^{-4}\mathrm{eV}^4 \qquad (30 \text{ to } 100 \text{ kpc})$$

The axion background provides a very diffuse concentration of a pseudoscalar condensate. Axions affect photons in an universal way. How could it be detected ?

I will discuss three effects:

1.- Cold axions influence cosmic ray propagation.

2.- Some photon wave-lengths are forbidden in the universe.

3.- Cold axions induce an additional rotation in the polarization plane (not the usual one!).

Manifest breaking of Lorentz symmetry

Let us consider (for the moment just as a theoretical possibility) the possibility of explicit breaking of Lorentz breaking by means of a time-like constant axial vector. Consider electromagnetism in such a background

$$\mathcal{L} = \mathcal{L}_{\rm INV} + \mathcal{L}_{\rm LIV}$$

$$\mathcal{L}_{\rm INV} = -\,\tfrac{1}{4}\,F^{\alpha\beta}F_{\alpha\beta} + \bar\psi[\not\partial - e\,\not A - m_e]\psi \qquad \mathcal{L}_{\rm LIV} = \tfrac{1}{2}\,m_\gamma^2\,A_\mu\,A^\mu + \,\tfrac{1}{2}\,\eta_\alpha A_\beta \widetilde{F}^{\,\alpha\beta}$$

It will be useful for us to keep $m_\gamma > 0$. Otherwise gauge invariance is manifest.

E.o.M.:

$$\left\{g^{\,\lambda\nu}\left(k^2 - m_\gamma^2\right) + i\,\varepsilon^{\,\lambda\nu\alpha\beta}\,\eta_\alpha\,k_\beta\right\}\tilde{A}_\lambda(k) = 0$$

We can build two complex and space-like chiral polarization vectors $\varepsilon^\mu_\pm(k)$ which satisfy the orthonormality relations

$$-\,g_{\,\mu\nu}\,\varepsilon^{\mu\,*}_{\pm}(k)\,\varepsilon^{\nu}_{\pm}(k) = 1 \qquad g_{\,\mu\nu}\,\varepsilon^{\mu\,*}_{\pm}(k)\,\varepsilon^{\nu}_{\mp}(k) = 0$$

In addition we have

$$\varepsilon^\mu_T(k) \sim k^\mu \qquad \varepsilon^\mu_L(k) \sim k^2\eta^\mu - k^\mu\eta\cdot k$$

They fulfill

$$g_{\,\mu\nu}\,\varepsilon^{\mu\,*}_{A}(k)\,\varepsilon^{\nu}_{B}(k) \,=\, g_{\,AB} \qquad g^{\,AB}\,\varepsilon^{\mu\,*}_{A}(k)\,\varepsilon^{\nu}_{B}(k) \,=\, g^{\,\mu\nu}$$

Different physics in different frames

Let us now assume that $\eta_\alpha = \partial_\alpha a(t) = \eta \delta_{\alpha 0}$

The polarization vectors of positive and negative chirality are solutions of the vector field equations if and only if

$$k_\pm^\mu = (\omega_{\mathbf{k}\,\pm}, \mathbf{k}) \qquad \omega_{\mathbf{k}\,\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta|\mathbf{k}|} \qquad \varepsilon_\pm^\mu(\mathbf{k}, \eta) = \varepsilon_\pm^\mu(k_\pm) \quad \left(k_\pm^0 = \omega_{\mathbf{k}\pm} \right)$$

In order to avoid problems with causality we want $k_\pm^2 > 0$. For photons of negative chirality this happens iff

$$|\mathbf{k}| < \frac{m_\gamma^2}{\eta} \equiv \Lambda_\gamma$$

for $m_\gamma = 0$ they cannot exist ($\gamma \to \gamma\gamma\gamma$).

As is known to everyone the processes $e^- \to e^-\gamma$ or $\gamma \to e^+e^-$ cannot occur. However here physics is different in different frames and for the latter process

$$\omega_{\,\mathbf{k}\,\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta\,|\mathbf{k}|} = \sqrt{\mathbf{p}^2 + m_e^2} + \sqrt{(\mathbf{p} - \mathbf{k})^2 + m_e^2}$$

Possible iff

$$|\mathbf{k}| \geq \frac{4m_e^2}{\eta} \equiv k_{\,\mathrm{th}} \qquad (m_\gamma = 0)$$

The electron-positron pairs will be created with a large momentum.

Lorentz violation from axions

Axion-photon coupling:

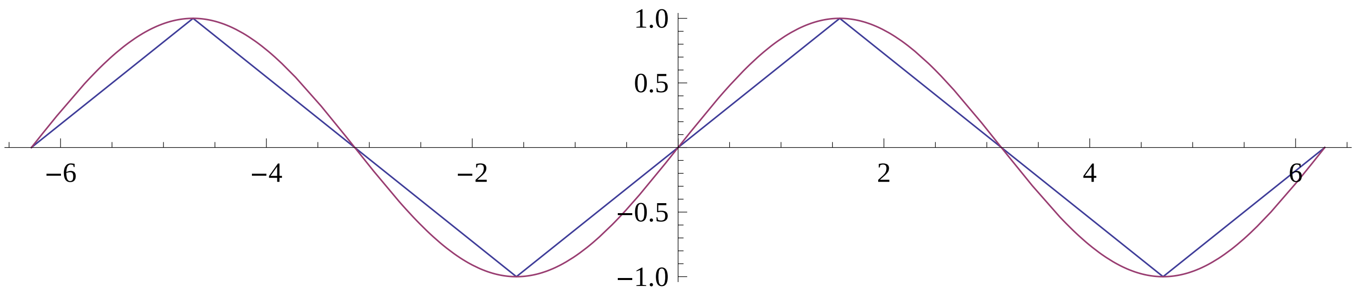
$$\Delta\mathcal{L} = -g_{a\gamma\gamma}\frac{\alpha}{\pi}\frac{a_0}{f_a}\cos(m_a t)\,\epsilon^{ijk}A_i F_{jk}$$

Popular models such as DFSZ and KSVZ all give $g_{a\gamma\gamma} \simeq 1$.

If all momenta involved are large $\mathbf{k} >> m_a$ it makes sense to treat the axion background adiabatically with a (quasiconstant) derivative

$$\Delta\mathcal{L} = \frac{1}{4}\eta\epsilon^{ijk}A_i F_{jk} = \frac{1}{2}\eta_\alpha A_\beta \tilde{F}^{\alpha\beta}$$

with $\eta_\alpha = (\eta, 0, 0, 0)$



The situation when $\mathbf{k} \leq m_a$ will be discussed later.

Discussion set-up

Astrophysical bounds:

$$f_a > \mathcal{O}(10^{11})\ \mathrm{GeV}, \qquad 10^{-3}\mathrm{eV} > m_a > 10^{-6}\ \mathrm{eV} \Rightarrow |\eta| \simeq \alpha\frac{\sqrt{\rho_a}}{f_a} \simeq 10^{-23} - 10^{-24}\mathrm{eV}$$

Direct bounds are weaker:

$$\eta < 10^{-20}\mathrm{eV}$$

η is the relevant quantity for all the effects discussed in this talk.

All the considerations in this presentation refer to vacuum propagation, i.e. we take $m_\gamma = 0$. Everything is computed at tree level in QED but non-linearities such as the ones described by the Euler-Heisenberg effective lagrangian can be easily included (e.g. Karbstein talk).

a-Bremsstrahlung in cosmic rays

Processes such as $p \to p\gamma$ or $e \to e\gamma$ are possible in LIV QED.

$$p(\mathbf{p}) \to p(\mathbf{p}-\mathbf{k})\gamma(\mathbf{k})$$

Energy conservation:

$$\sqrt{E^2 + k^2 - 2pk\cos\theta} + \sqrt{k^2 \pm \eta k + m_\gamma^2} - E = 0, \qquad \eta > 0$$

Kinematical constraints:

We consider the case $m_\gamma = 0$ (but note that η is very small). The kinematic limits are

$$p_{th} = 0$$

$$k_{min} = \eta, \quad \text{for} \cos\theta = -\eta/2p$$

$$k_{max} = \frac{E^2}{p + \frac{m_p^2}{\eta}}, \quad \text{for} \cos\theta = 1$$

$k_{max} \simeq E$ for $E \gg m_p^2/|\eta|$ $\qquad$ $k_{max} \simeq |\eta|E^2/m_p^2$ for $E \ll m_p^2/|\eta| \Rightarrow k < \frac{|\eta|E^2}{m_p^2}$

Energy loss and attenuation length

The differential decay width will be

$$d\Gamma(Q) = \frac{\alpha}{2}\frac{|\mathbf{k}|}{|\mathbf{p}|}\frac{1}{E\omega_{\mathbf{k}}}(-p\cdot k + |\mathbf{p}|^2\sin^2\theta)d|\mathbf{k}|$$

The rate of energy loss will be

$$\frac{dE}{dx} = -\frac{1}{v}\int d\Gamma(Q)w(Q)$$

$$\frac{dE}{dx} = -\frac{\alpha}{2}\frac{1}{p^2}\int kdk[-\frac{1}{2}(m_\gamma^2 + \eta k) + p^2(1-\cos^2\theta)]$$

There are two relevant limits

$$E \ll \frac{m_p^2}{|\eta|} \longrightarrow \frac{dE}{dx} = -\frac{\alpha\eta^2 E^2}{4m_p^2}.$$

$$E \gg \frac{m_p^2}{|\eta|} \longrightarrow \frac{dE}{dx} = -\frac{\alpha|\eta|}{3}E$$

The axion (very weak) shield

There are two key scales in this problem

$$E_{th} \simeq 2m_\gamma m_p/\eta \qquad \text{and} \qquad m_p^2/\eta$$

If $E \gg m_p^2/|\eta|$

$$E(x) = \exp - \frac{\alpha|\eta|}{3} x$$

For reasonable η this would give a mean free path in the range $\mathcal{O}(1)$ to $\mathcal{O}(10)$ kpc. This would imply that cold axions act as a powerful shield against very energetic cosmic rays. The detection of cosmic rays above that energy would impose a rather stringent bound on η.

However, this is not so because even for the most energetic cosmics, just below the GZK cut-off of 10^{20} eV, we are well below the cross-over scale $m_p^2/|\eta|$. In this regime the expression for $E(x)$ is

$$E(x) = \frac{E(0)}{1 + \frac{\alpha\eta^2}{4m_p^2} E(0)x}$$

For extremely large distances $E(x) \sim \frac{1}{x}$,

From the likely detection of extragalactic cosmic rays we get a model independent bound

$$\eta < 10^{-15}\ \text{eV} \Rightarrow f_a > 100\ \text{GeV}$$

(i.e. exclude weak scale axions in a completely model independent way)

Axion-induced radioemission

Spectrum of emission (per unit time)

$$\int_{E_{min}}^{E_{GZK}} dE\, n(E) \frac{d\Gamma}{dk} \qquad E_{min} = \sqrt{\frac{m^2 k}{\eta}} > E_{th} \qquad E_{th} = 2\frac{m_{p,e} m_\gamma}{|\eta|} \simeq 0$$

E_{th} must at least be 1 GeV to exclude solar electrons.

The flux of photons is

$$\frac{d^3 N_\gamma}{dk dS dt} = \int_{E_{\min}(k) > E_{th}}^{\infty} dE\ t(E) J(E) \frac{d\Gamma(E,k)}{dk}\ , \quad E_{min}(k) = \sqrt{\frac{m^2 k}{\eta}}, \quad E_{th} = 2\frac{m m_\gamma}{\eta}$$

$t(E)$ is approximately constant: $t(E) \approx T_p = 10^7$ yr for protons.

$t(E) \approx T_e = 5 \cdot 10^5$ yr for electrons in average, but it is not constant: $t(E) \sim 1/E$

The photon energy flux is obtained by multiplying the photon flux by the energy of a photon with momentum k:

$$I(k) = \omega(k) \int_{E_{min}(k) > E_{th}}^{\infty} dE\ t(E) J(E) \frac{d\Gamma}{dk}$$

The integral is dominanted by the end point E_{min}.

Axion-induced radioemission

Radiation flux intensity

$$I^p_\gamma(k) \simeq \frac{\alpha\eta T}{2}\frac{J_p(E_{min}(k))E_{min}(k)}{\gamma_{min}-1}$$

$$I^e_\gamma(k) \simeq \frac{\alpha\eta T_0}{2}\frac{J_e(E_{min}(k))}{\gamma_{min}}$$

Energies are all expressed in eV. The value γ_{min} is determined by the cosmic ray flux in a given range of E.

The dominant contribution comes from electrons

$$I^e_\gamma(k) \simeq 3\times 10^2 \times \left(\frac{\eta}{10^{-20}\ \mathrm{eV}}\right)^{2.52}\left(\frac{k}{10^{-7}\ \mathrm{eV}}\right)^{-1.52}\ \mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$$

($\times 100$ if proton normalization is used)

For protons

$$I^p_\gamma(k) \simeq 6\times\left(\frac{T}{10^7\ \mathrm{yr}}\right)\left(\frac{\eta}{10^{-20}\ \mathrm{eV}}\right)^{1.84}\left(\frac{k}{10^{-7}\ \mathrm{eV}}\right)^{-0.84}\ \mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$$

Signal is ~ 1 mJy at best

1 Jy = 10^{-26} W Hz^{-1} m^{-2} sr^{-1} $\simeq 1.5\times 10^7$ eV eV^{-1} m^{-2} s^{-1} sr^{-1}

Axion-induced radioemission

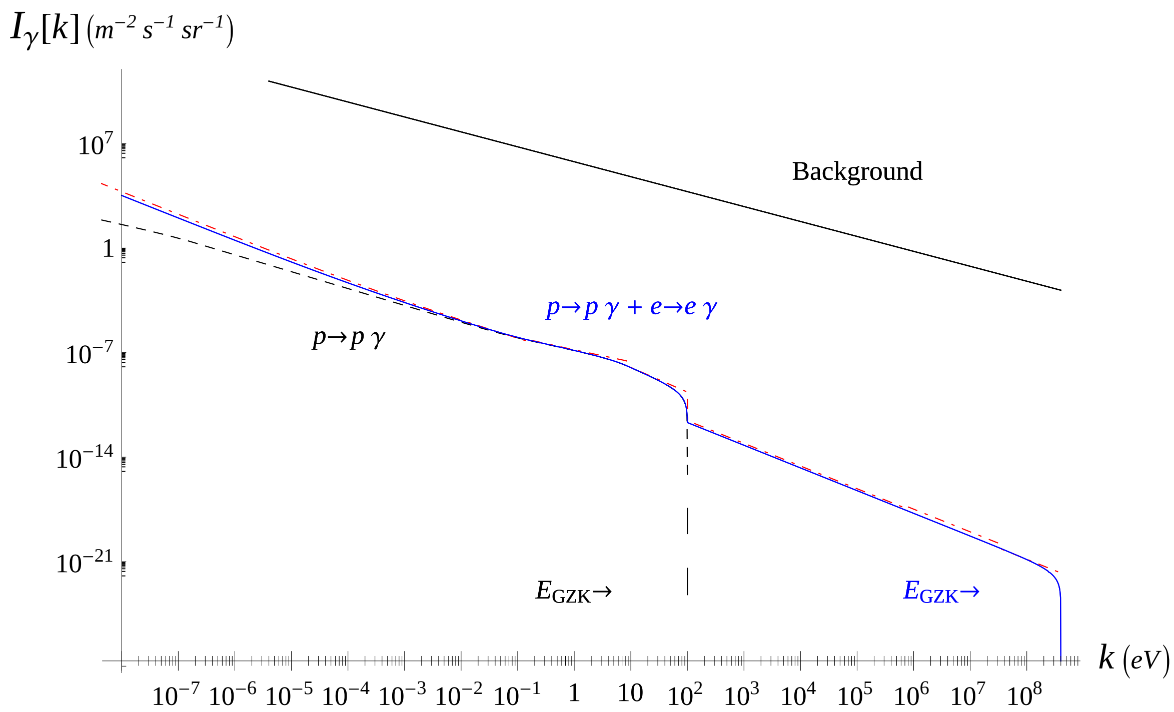


Figure 1: Energy radiated as a function of the wave vector using the most conservative hypothesis.

Antennas

LWA

- New Mexico, being deployed. Sensitivity down to 30 MHz and 10^{-4} Jy

10^{-4} Jy $\simeq 10^3$ m^{-2} s^{-1} sr^{-1}

maybe even less depending on extension

SKA

- Australia, under construction. Sensitivity down to 70 MHz and 650 nJy
  at the lowest frequency assuming an integration time of 50hrs

Far side of the Moon

- Not limited by atmosphere opacity
- Designs exist (ESA) reporting sensitivities down to 10^{-5} Jy

Sensitivity of antennas is not an issue, but the background is a tough enemy.

Galactic noise

The window $\lambda = 10$ cm (3 GHz) to $\lambda = 100$ m (30 MHz) corresponds to 10^{-5} eV to 10^{-8} eV.

This region has a strong background from galactic noise from synchrotron radiation.

In the 100 MHz region we expect the signal to be six orders of magnitude below the background on average.

However

— Sensitivity of planned antennas may be as low as $10^{-12}\times$ background

— Galactic magnetic field $\mathbf{H}$ varies by many orders of magnitude and the background $\sim \mathbf{H}^2$

— The power dependence of the electron yield is different from the SR in the galactic plane

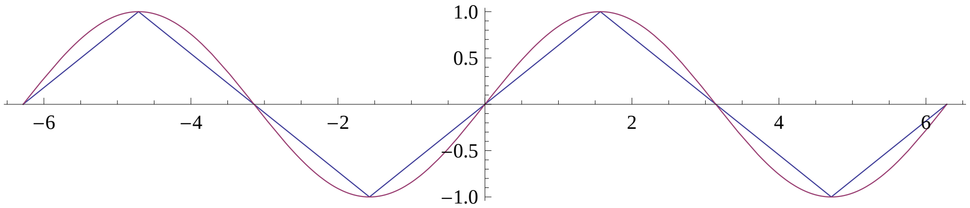
— Regions of low magnetic field and high galactic latitude are to be explored

— Polarization of the radiation is different

— A magnetic field actually enhances the effect

What if $|k| \le m_a$?

$a(t)$ changes sign with a period $2\pi/m_a$ and this is now relevant. Let us approximate the sinusoidal variation and solve exactly for the propagating modes



The equation for $\hat{A}_\nu(t,\vec{k})$ is

$$\left[g^{\mu\nu}(\partial_t^2+\vec{k}^2)-i\epsilon^{\mu\nu\alpha\beta}\eta_\alpha k_\beta\right]\hat{A}_\nu(t,\vec{k})=0. \qquad \hat{A}_\nu(t,\vec{k})=\sum_{\lambda=+,-} f_\lambda(t)\varepsilon_\nu(\vec{k},\lambda).$$

We now write $f(t)=e^{-i\omega t}g(t)$ and demand that $g(t)$ have the same periodicity as $\eta(t)$. This requires

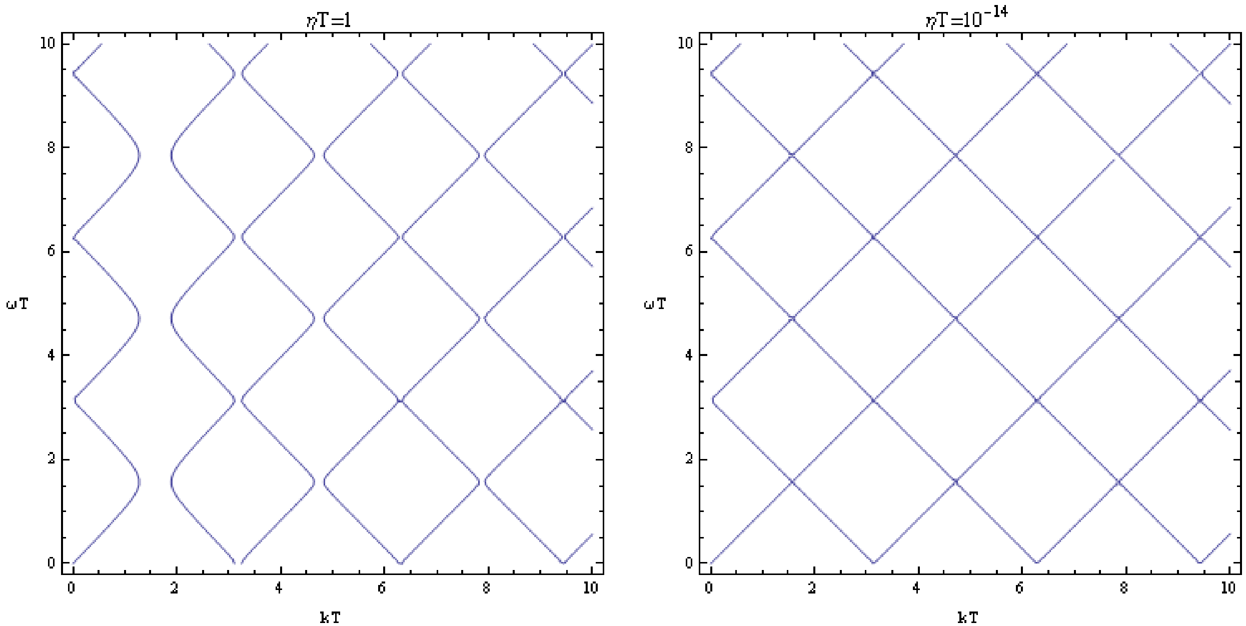
$$\cos(2\omega T)=\cos(\alpha T)\cos(\beta T)-\frac{\alpha^2+\beta^2}{2\alpha\beta}\sin(\alpha T)\sin(\beta T), \quad T=\frac{\pi}{M_a}, \quad \alpha,\beta=\sqrt{k^2\pm\eta_0 k}$$

For $\eta_0\equiv\eta_{\max}\ll m_a$ no relevant variation wrt the decay rate computed assuming $|\mathbf{k}|>>m_a$ is found, and this can be understood intuitively

Forbidden wavelengths

However if η_0/m_a grows there is a surprise ($\eta_0 = 2g_{a\gamma\gamma}\frac{\alpha}{\pi}\frac{a_0 m}{f_a}$)

$$\cos(2\omega T) = \cos(\alpha T)\cos(\beta T) - \frac{\alpha^2+\beta^2}{2\alpha\beta}\sin(\alpha T)\sin(\beta T)$$



Some photon wavelengths are forbidden in the universe if there is a cold axion background.

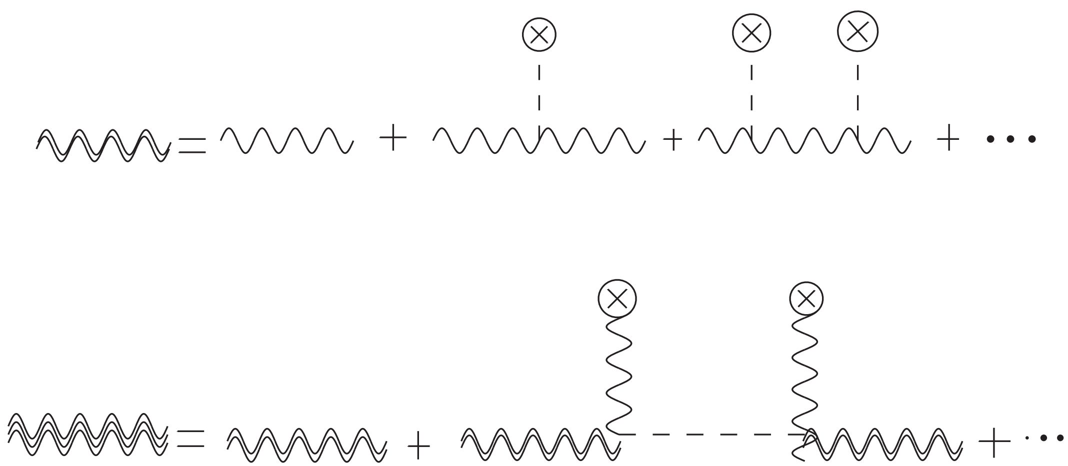
Can this be seen in table-top experiments?

Exotic rotation of the polarization plane

For this we need to find the photon propagator in two separate backgrounds

— A constant magnetic field (well kown result). For simplicity we shall assume $\mathbf{k} \cdot \vec{B} = 0$.

— The cold axion background (new)



Exotic rotation of the polarization plane

The photon propagator for $\mathbf{k} \cdot \vec{B} = 0$ is ($\vec{b} \equiv \frac{2 g_{a\gamma\gamma} \alpha}{\pi f_a} \vec{B}$)

$$\mathcal{D}_{\mu\nu}(k) \simeq \frac{-i g_{\mu\nu}}{k^2} + \frac{i k_0^2 b_\mu b_\nu}{k^2 [k^2 (k^2 - m^2) - k_0^2 \vec{b}^2]} - g_\mu^j g_\nu^l \frac{\eta_0 k_0^2 \left[b_j (\vec{b} \times \vec{k})_l - b_l (\vec{b} \times \vec{k})_j \right]}{k^4 \left[k^2 (k^2 - m^2) - k_0^2 \vec{b}^2 \right]}$$

Then for a photon plane wave initially with an electric field forming an angle β with $\vec{B}$ the plane of polarization rotates as

$$\tan 2\alpha(x) = \frac{[1 + 2f(x)] \sin 2\beta + 3\eta_0 |x| \cos 2\beta}{4f(x) + [1 + 4f(x)] \cos 2\beta - 3\eta_0 |x| \sin 2\beta},$$

where

$$f(x) = \frac{\vec{b}^4}{16 m^4} k_0^2 |x|^2,$$

and the expected value for the angle is

$$\bar{\alpha} = -\frac{1}{2} \frac{[1 + 2f(x)] \sin 2\beta + 3\eta_0 |x| \cos 2\beta}{[1 + 4f(x)] + 4f(x) \cos 2\beta}.$$

Exotic rotation of the polarization plane

$$\tan 2\alpha(x) = \frac{[1+2f(x)]\sin 2\beta + 3\eta_0|x|\cos 2\beta}{4f(x)+[1+4f(x)]\cos 2\beta - 3\eta_0|x|\sin 2\beta},$$

Surprises:

— The rotation survives even without magnetic field.

— The effect is independent of the frequency.

Warning: the previous result hold only for table-top experiments only, when the photon can approximately be considered an eigenstate of energy.

Summary

Propagation of photons, electrons, protons,... in a pseudoscalar background is well described by a LIV version of QED sometimes termed Chern-Simons QED. There are no hidden assumptions or model dependences of any kind in the predictions.

Properties are rather unfamiliar.

The dispersion relation is modified and this makes possible processes such as $\gamma \to e^+e^-$ or $p \to p\gamma$, not unlikely the Cerenkov effect.

A background of cold axions can be described in this way and it has unexpected consequences on cosmic ray propagation. Cosmic rays radiate circularly polarized light at a low rate.

Although small, the emitted radiation falls well within the sensitivity of current instruments, but the background is a problem. Could it be circumvented?

Photons in the universe have not well defined frequencies and some wave lengths are forbidden.

There is a small rotation in the polarization plane of photons with rather strange properties.

Effective photon mass

Effective photon mass:

$$m_\gamma^2 = 4\pi\alpha \frac{n_e}{m_e}, \quad n_e = 10^{-7}\text{c}m^{-3}$$

This number is of the order of 10^{-15} eV, but we shall assume the more conservative limit 10^{-18} eV, compatible with some astrophysical bounds in the 10^{-16} to 10^{-17} eV region. However, for the energies of a cosmic ray ($|\mathbf{k}| >> 1/l$, l = mean free path) the photon mass can be assumed to be exactly zero.

Kinematical constraints for $m_\gamma > 0$

Let us now consider $m_\gamma > 0$

$$p_{th} \simeq \frac{2 m_\gamma m_p}{\eta}$$

$$k(\theta_{max}) \simeq \frac{2m_\gamma^2}{\eta}(1 - 3\frac{pm_\gamma^2}{E^2\eta}) \stackrel{p>>p_{th}}{\longrightarrow} \frac{2m_\gamma^2}{\eta}, \qquad \sin^2\theta_{max} \to \frac{\eta^2}{4m_\gamma^2}$$

θ_{max} is small, photons are emitted in a narrow cone

In the opposite extreme, for zero angle there are two solutions

$$k_+(0) \simeq \frac{E^2\eta + pm_\gamma^2 + E\sqrt{E^2\eta^2 - 4m_p^2 m_\gamma^2 + 2p\eta m_\gamma^2}}{2p\eta + 2m_p^2} \stackrel{p>>p_{th}}{\longrightarrow} \frac{E^2}{p + \frac{m_p^2}{\eta}}$$

which is the same result obtained before, and

$$k_-(0) \simeq \frac{E^2\eta + pm_\gamma^2 - E\sqrt{E^2\eta^2 - 4m_p^2 m_\gamma^2 + 2p\eta m_\gamma^2}}{2p\eta + 2m_p^2} \stackrel{p>>p_{th}}{\longrightarrow} \frac{m_\gamma^2}{\eta}$$

$$k_-(0) < k(\theta_{max}) < k_+(0)$$

Kinematical constraints for $m_\gamma > 0$

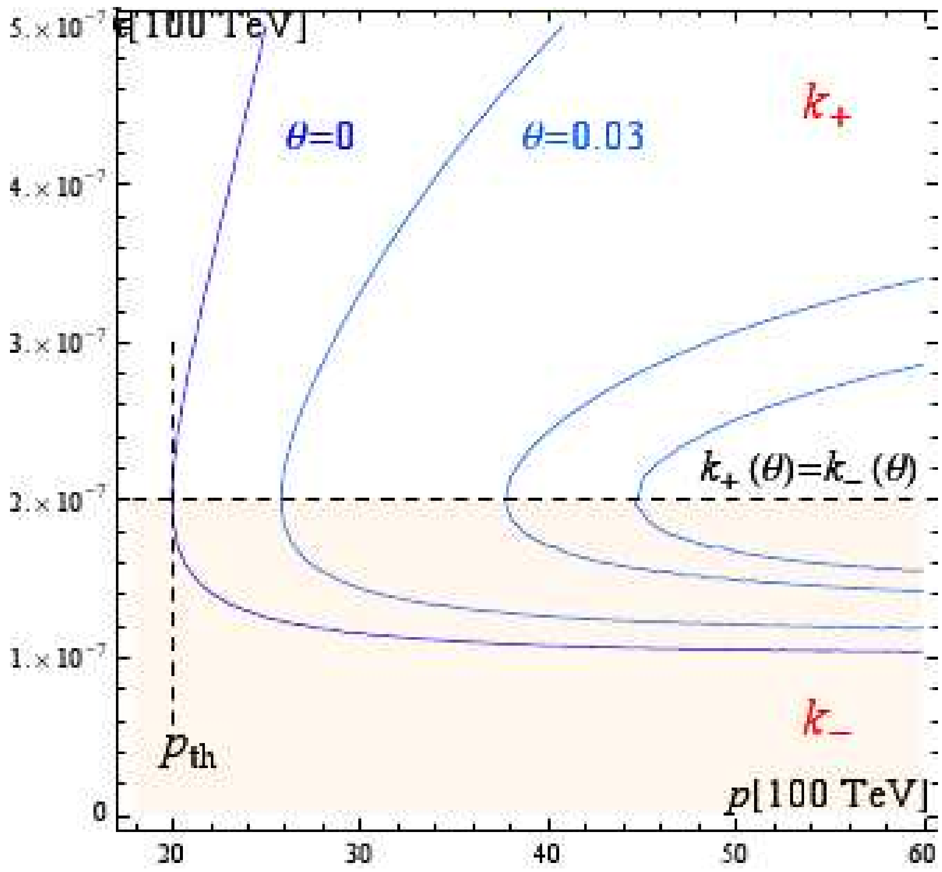


Figure 2: The solutions $k_\pm$ of the energy conservation equation for $m_\gamma = 10^{-18}$ eV and $\eta = 10^{-24}$ eV.

Cosmic ray fluxes

Broadcasters:

Proton primaries

$$n(E) = N \times \begin{cases} E^{-2.68} & 10^9 \leq E \leq 4 \cdot 10^{15} \\ 1.12 \cdot 10^{19} E^{-3.26} & 4 \cdot 10^{15} \leq E \leq 4 \cdot 10^{18} \\ 3.85 \cdot 10^{-4} E^{-2.59} & 4 \cdot 10^{18} \leq E \leq 2.9 \cdot 10^{19} \\ 7.34 \cdot 10^{29} E^{-4.3} & E \geq 2.9 \cdot 10^{19} \end{cases}$$

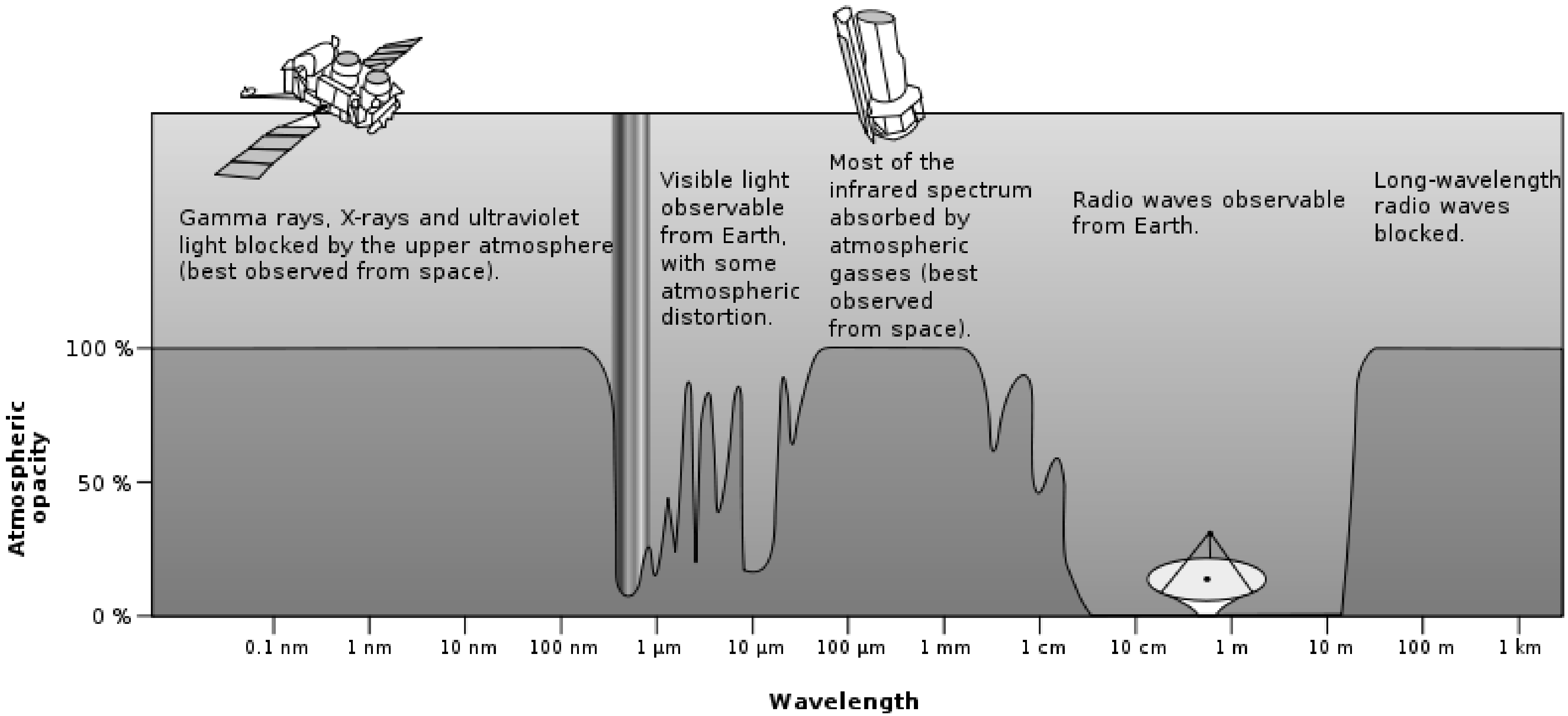
Electron (+positrons) primaries

$$n(E) = N \times \begin{cases} 0.01 E^{-2.68} & E \leq 5 \cdot 10^{10} \\ 71.1 E^{-3.04} & E \geq 5 \cdot 10^{10} \end{cases}$$

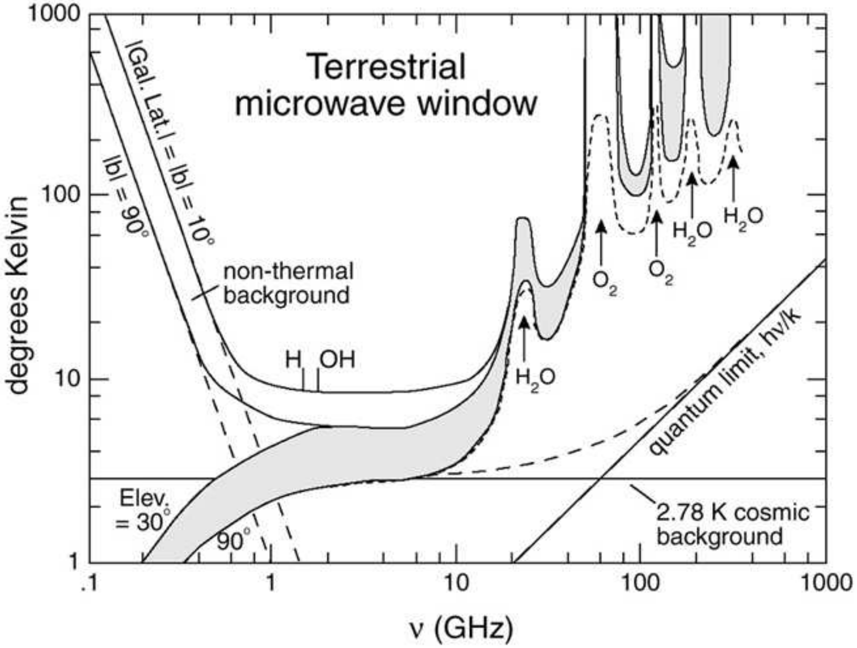
Units: eV^{-1} m^{-2} s^{-1} sr^{-1}.

Need to assume a given function $t(E)$ and combine with isotropy hypothesis to find the photon yield.

Atmosphere opacity



Galactic noise



Unit in radio astronomy:

1 Jy = 10^{-26} W Hz^{-1} m^{-2} sr^{-1} $\simeq 1.5 \times 10^7$ eV eV^{-1} m^{-2} s^{-1} sr^{-1}