

# Z' from GUTs, weak CP, strong CP, axions, and the $\mu$ problem,

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What can be there beyond SM?  
New CP? Axions? SUSY? String?

1. Is there  $U(1)'$  ?
2. The weak CP violation
3. The  $\mu$  problem



# 1. Is there $U(1)'$ beyond SM?

In the SM, the  $P$  violation in weak interactions is ultimately given at low energy perspective by the Glashow-Salam-Weinberg chiral model of weak interactions.



Kim-Shin, arXiv:1104.5500

“Z’ from  $SU(6) \times SU(2)_h$  GUT,  $W_{jj}$  anomaly  
and Higgs boson mass bound”

1. No-go theorem for  $U(1)_B$  from  $E_6$ .
2. If Z’ found below 10 TeV, our understanding of the SM from subgroups of  $E_6$  is not realized.

GUTs,  $SU(5)$ ,  $SO(10)$ ,  $SU(3) \times SU(3) \times SU(3)$ ,  
 $SU(6) \times SU(2)$ , flipped  $SU(5)$  are all not enough.

This is independent of SUSY.



# SU(6)xSU(2) model

JEK, PLB 107, 69 (1982),  
 JEK, PLB 656, 207 (2007) [arXiv: 0707.3292],  
 K.-S. Choi and JEK, PRD 83, 065016 (2011)  
 JEK and S. Shin, arXiv:1104.5500.

$$15_L \equiv (15, 1) = \begin{pmatrix} 0 & u^c & -u^c & u & d & D \\ -u^c & 0 & u^c & u & d & D \\ u^c & -u^c & 0 & u & d & D \\ -u & -u & -u & 0 & e^c & H_u^+ \\ -d & -d & -d & -e^c & 0 & H_u^0 \\ -D & -D & -D & -H_u^+ & -H_u^0 & 0 \end{pmatrix},$$

$$\bar{6}_{2,1} \equiv (\bar{6}, 2^\uparrow) = \begin{pmatrix} d^c \\ d^c \\ d^c \\ -\nu_e \\ e \\ N \end{pmatrix}, \quad \bar{6}_{2,2} \equiv (\bar{6}, 2^\downarrow) = \begin{pmatrix} D^c \\ D^c \\ D^c \\ -H_d^0 \\ H_d^- \\ N' \end{pmatrix}. \quad (1)$$

$$27 = (15, 1) + (6^*, 2)$$

For diagonal subgroups of  $E_6$ , any  $U(1)$  generator can be a linear combination of Cartan subgroup of  $E_6$ . So, we prove in terms of the Cartan subgroup of  $SU(6) \times SU(2)$ .

$$F_3, F_8, T_3, Y, Y_6, X_3$$

Leptons and Higgs doublets do not carry the baryon number.

$$B = aY + bY_6 + cX_3 + dR$$

$$\begin{aligned} e^c : & \quad a - \frac{1}{3}b + (R_{15} - 1)d = 0 \\ (\nu, e) : & \quad -\frac{1}{2}a + \frac{1}{6}b + \frac{1}{2}c + (R_{\bar{6}} - 1)d = 0 \\ H_d : & \quad -\frac{1}{2}a + \frac{1}{6}b - \frac{1}{2}c + R_{\bar{6}}d = 0 \\ H_u : & \quad +\frac{1}{2}a + \frac{2}{3}b + R_{15}d = 0 \end{aligned}$$

No solution.



For leptophobic  $Z'$ , we may try

$$\begin{aligned}
 e^c : & \quad a - \frac{1}{3}b + (R_{15} - 1)d = 0 \\
 (\nu, e) : & \quad -\frac{1}{2}a + \frac{1}{6}b + \frac{1}{2}c + (R_{\bar{6}} - 1)d = 0 \\
 H_d : & \quad -\frac{1}{2}a + \frac{1}{6}b - \frac{1}{2}c + R_{\bar{6}}d = 0 \\
 H_u : & \quad +\frac{1}{2}a + \frac{2}{3}b + R_{15}d = 0
 \end{aligned}$$

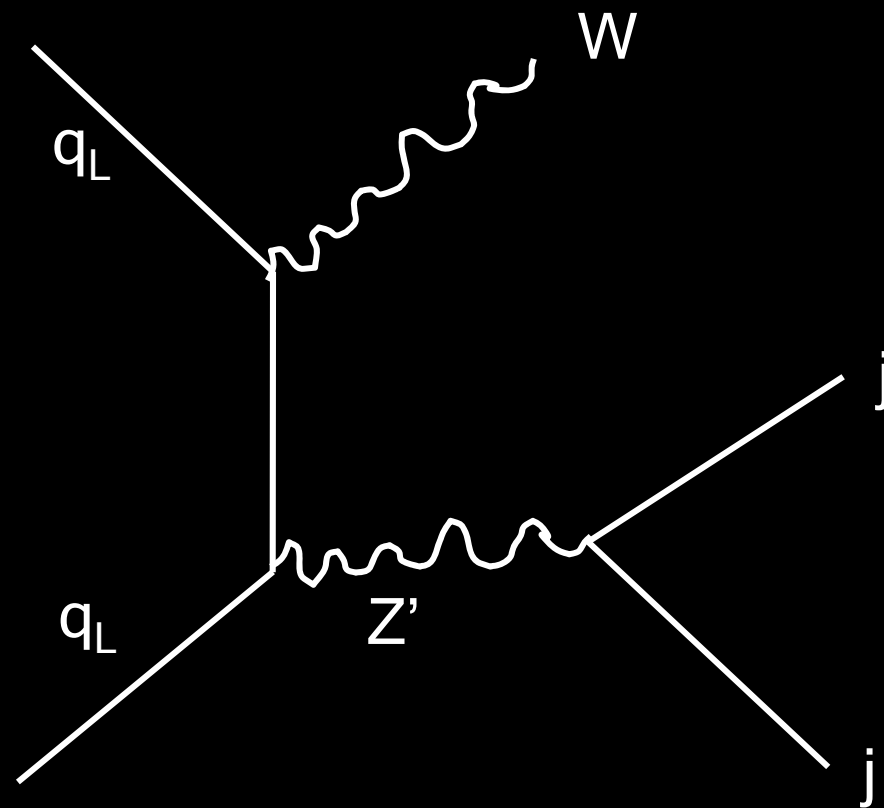
$$Y' = Y_6 - \frac{1}{3}Y = \frac{5}{6} \left( \frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, 0, 0, 1 \right)$$

$H_u$  carries a nonvanishing  $Y'$ . So,  $N$  has a nonvanishing  $Y'$  and singlet neutrino mass scale is the  $Z'$  mass scale.

So, we consider  $Z'$  coupling both to  $B$  and  $L$ .

For  $Z_6$  hexality, we consider  $SU(6) \times SU(2)$ .

The  $W_{jj}$  anomaly may arise from





Still, we studied the  $Z$ - $Z'$  mass in the  $SU(6) \times SU(2)$  model with fine tuned coupling constants.

So, we consider  $SU(6) \times SU(2)$

In this study, we assume of course the lepton coupling to  $Z'$ . Then, the LEP2 precision experiment bound on the rho parameter is crucial to constrain the model.

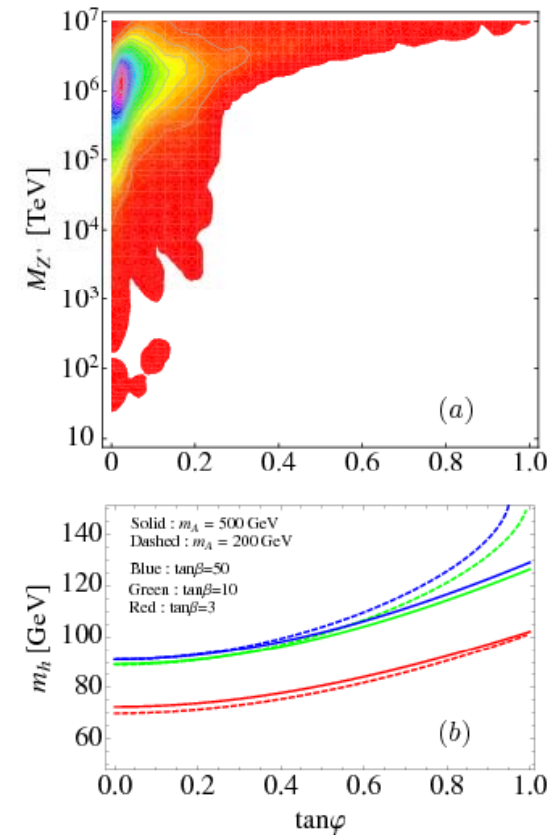


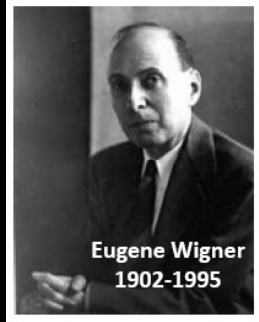
FIG. 1. Masses of (a)  $Z'$  and (b) the lightest CP even Higgs  $m_h$  as functions of  $\tan\varphi$ . The total number of data points is 14, 235, and the region with no data points is white. From the red color, the colors are separated by the density of points, increment of ten for each step.

## 2. The weak CP problem

The charge conjugation C and parity P have been known as exact symmetries in atomic physics, i.e. in electromagnetic interactions.



1924: Atomic wave functions are either symmetric or antisymmetric:  
Laporte rule



1927: Nature is parity symmetric, Wigner:  
Laporte rule = parity symmetric

Quantum mechanics was developed after the atomic rule of Laporte was known. It is based on the

## SYMMETRY PRINCIPLE !!!!

In QM, these symmetry operations are represented by unitary operators. For continuous symmetries, we represent them by generators

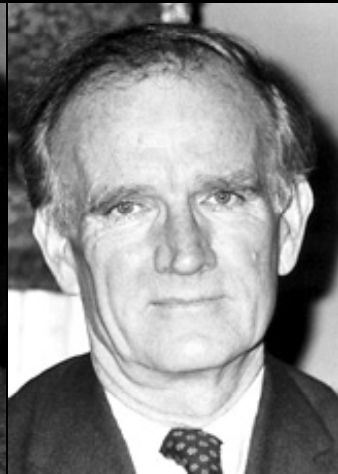
$$U = e^{i\theta \cdot F}$$

where  $F$  is a set of generators.

For discrete symmetries, we use  $U$  directly like  $P$ ,  $C$ ,  $CP$ , etc.



CP violation observed in the neutral K-meson system (and now from B-meson system) needed to introduce a CP violation in the SM. It was given by the Kobayashi-Maskawa model.



The CKM matrix has been written by many since the KM paper,

Kobayashi-Maskawa, Prog. Theor. Phys. 49 (1973) 652  
using N. Cabibbo, PRL 10 (1963) 531

Maiani, PLB 62 (1976) 183

Chau-Keung, PRL 53 (1984) 1802

Wolfenstein, PRL 51 (1983) 1945 : Approximate form

Qin-Ma, PLB 695 (2011) 194 : Approximate form

**Recently, Seo and I wrote an exact CKM matrix  
replacing the Wolfenstein form. Another complication in lit.?  
or reaching to the end of the road of writing the CKM matrix?**

CP violation books contain basics:

- G. C. Branco, L. Lavoura and J. P. Silva,  
CP Violation, Int. Ser. Monogr. Phys 103 (1999).  
I. I. Bigi and A. I. Sanda, CP violation, Cambridge  
Monographs on Particle Phys. and Cosmology (2009)

Still, I would like to repeat the (probably) knowns about  
the CKM matrix  $V(\text{CKM})$ :

1.  $\text{Det. } V(\text{CKM})$  is better to be real !
2.  $3 \times 3$   $V(\text{CKM})$  is complex to describe CP violation
3. If any among 9 elements is zero, then there is no weak CP violation.
4.  $\lambda$  is a good expansion parameter (Wolfenstein) .
5.  $(31) \cdot (22) \cdot (13)$  is the barometer of weak CP violation.
6. Eventually,  $V(\text{CKM})$  is derivable from the Yukawa texture.

# 1. Det. $V(\text{CKM})$ is better to be real !

If not, then Arg. Det.  $M_q$  is not zero. Usually, we remove this to define a good quark basis. The PQ symmetry? Or calculable models?

KM model has a phase. MCK do not have a phase. If it has a phase, then

$$\begin{aligned} L &= L(\text{quark - gauge int.}) + L(\text{Higgs boson}) \\ &+ \frac{\theta}{32\pi^2} g_c^2 G\tilde{G} \quad \rightarrow \\ &= \dots + \frac{\theta - \delta}{32\pi^2} g_c^2 G\tilde{G} \end{aligned}$$

The neutron EDM has the problem. Fine-tuning. So, the CKM matrix having Det=0 is a good choice. But it is not absolutely necessary.

4.  $\lambda$  is a good expansion parameter (Wolfenstein) .

$$V_{\text{Wolf}} = \begin{pmatrix} 1 - \lambda^2/2, & \lambda, & A\lambda^3(\rho - i\eta) \\ -\lambda, & 1 - \lambda^2/2, & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta), & -A\lambda^2, & 1 \end{pmatrix}$$

$\lambda^3 \kappa_t e^{i\delta_t}$  (under  $A\lambda^3(1 - \rho - i\eta)$ )  
 $\lambda^3 \kappa_b e^{i\delta_b}$  (over  $A\lambda^3(\rho - i\eta)$ )  
 $+ \mathcal{O}(\lambda^4)$ .

We expand in terms of  $\theta_1$  since  $\theta_2$  and  $\theta_3$  are of order  $\theta_1^2$ .



Satisfying all the requirements, we write an exact CKM matrix,

$$V_{\text{KS}} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -c_2 s_1 & e^{-i\delta} s_2 s_3 + c_1 c_2 c_3 & -e^{-i\delta} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta} s_1 s_2 & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta} & c_2 c_3 + c_1 s_2 s_3 e^{i\delta} \end{pmatrix}$$

$$\begin{aligned} \theta_1 &= 13.0305^\circ \pm 0.0123^\circ = 0.227426 \pm 2.14, \\ \theta_2 &= 2.42338^\circ \pm 0.1705^\circ = 0.042296 \pm 2.976 \times 10^{-3}, \\ \theta_3 &= 1.54295^\circ \pm 0.1327^\circ = 0.027567 \pm 2.315 \times 10^{-3}, \\ \delta &= 89.0^\circ \pm 4.4^\circ. \end{aligned}$$

(31)(22)(13) is

$$-e^{i\delta} s_1^2 s_2 s_3 c_1 c_2 c_3 - s_1^2 s_2^2 s_3^2$$

The elements of Det. V(CKM) is,

$$\begin{aligned}
 V_{11}V_{22}V_{33} &= c_1^2 c_2^2 c_3^2 + c_1^2 s_2^2 s_3^2 + 2c_1 c_2 c_3 s_2 s_3 \cos \delta \\
 &\quad - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta} \\
 -V_{11}V_{23}V_{32} &= c_1^2 c_2^2 s_3^2 + c_1^2 s_2^2 c_3^2 - 2c_1 c_2 c_3 s_2 s_3 \cos \delta \\
 &\quad + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta} \\
 V_{12}V_{23}V_{31} &= s_1^2 s_2^2 c_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta} \\
 -V_{12}V_{21}V_{33} &= s_1^2 c_2^2 c_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta} \\
 V_{13}V_{21}V_{32} &= s_1^2 c_2^2 s_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta} \\
 -V_{13}V_{22}V_{31} &= s_1^2 s_2^2 s_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta} .
 \end{aligned}$$

All elements have the same imaginary part, due to our good choice of Det. being real. But, the individual part describes CP violating processes.

The approximate form is,

$$\begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} - \frac{\lambda^6}{16}(1 + 8\kappa_b^2), & \lambda, & \lambda^3 \kappa_b \left(1 + \frac{\lambda^2}{3}\right) \\ -\lambda + \frac{\lambda^5}{2}(\kappa_t^2 - \kappa_b^2), & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} - \frac{\lambda^6}{16} \\ & -\frac{\lambda^4}{2}(\kappa_t^2 + \kappa_b^2 - 2\kappa_b \kappa_t e^{-i\delta}) & \lambda^2 (\kappa_b - \kappa_t e^{-i\delta}) \\ & -\frac{\lambda^6}{12} (7\kappa_b^2 + \kappa_t^2 - 8\kappa_t \kappa_b e^{-i\delta}) & -\frac{\lambda^4}{6} (2\kappa_t e^{-i\delta} + \kappa_b) \\ -\lambda^3 \kappa_t e^{i\delta} \left(1 + \frac{\lambda^2}{3}\right), & -\lambda^2 (\kappa_b - \kappa_t e^{i\delta}) & 1 - \frac{\lambda^4}{2} (\kappa_t^2 + \kappa_b^2 - 2\kappa_b \kappa_t e^{i\delta}) \\ & -\frac{\lambda^4}{6} (2\kappa_b + \kappa_t e^{i\delta}) & -\frac{\lambda^6}{6} (2[\kappa_b^2 + \kappa_t^2] - \kappa_t \kappa_b e^{i\delta}) \end{pmatrix}$$

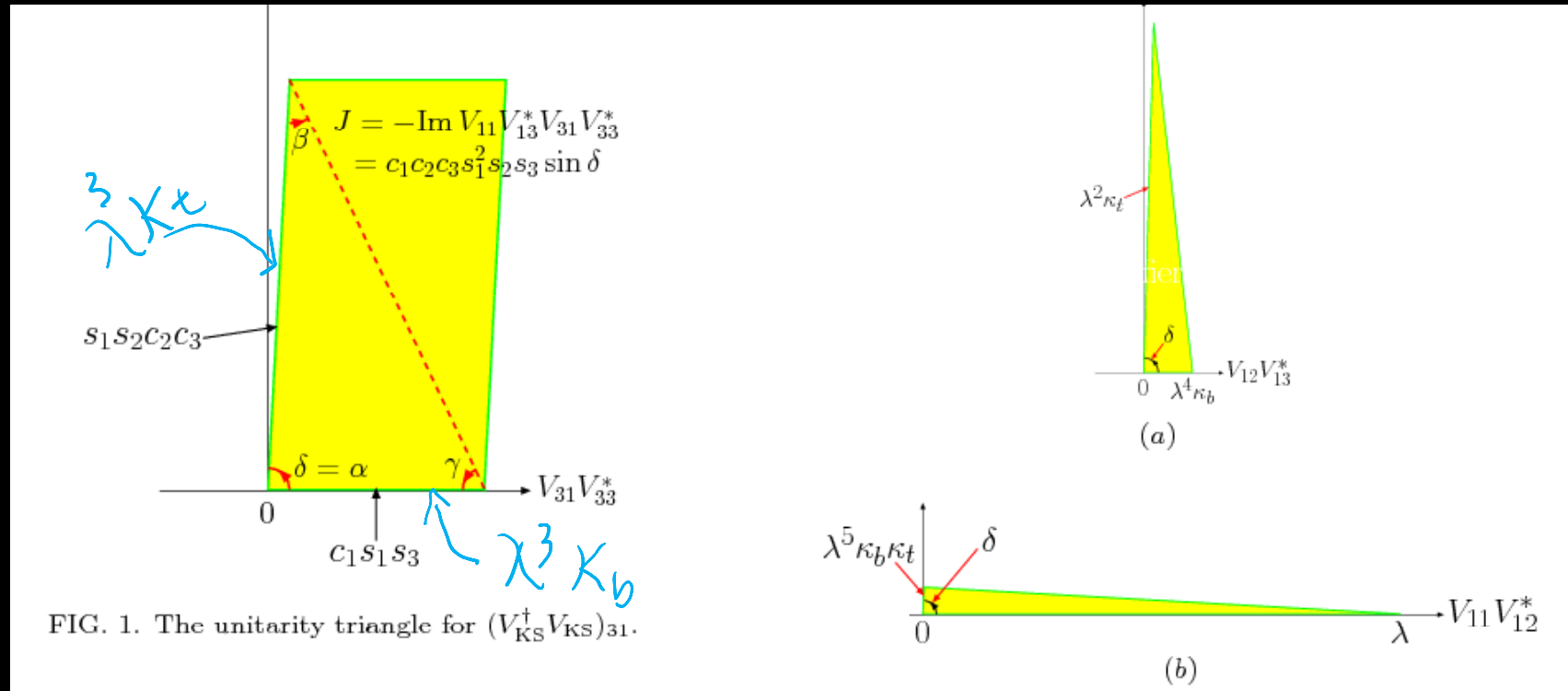
$$\lambda = 0.22527 \pm 0.00092,$$

$$\kappa_t = 0.7349 \pm 0.0141, \quad \kappa_b = 0.3833 \pm 0.0388,$$

$$\delta = 89.0^\circ \pm 4.4^\circ$$

$\kappa_b$ , or  $\kappa_t$ , or  $\delta$  being zero washes out the CP violation, in the exact or in the approximate form.

The Jarlskog triangles are



These can be read directly from  $V(\text{CKM})$ .

$$V_{\text{KS}} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -c_2 s_1 & e^{-i\delta} s_2 s_3 + c_1 c_2 c_3 & -e^{-i\delta} s_2 c_3 + c_1 c_2 s_3 \\ -e^{i\delta} s_1 s_2 & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta} & c_2 c_3 + c_1 s_2 s_3 e^{i\delta} \end{pmatrix}$$

Jarlskog removed the real parts by considering a commutator of the weak basis mass matrices.

$$C = -i \left[ L^{(u)+} M^{(u)} L^{(u)}, L^{(d)+} M^{(d)} L^{(d)} \right]$$

$$\text{Det.} C = i(e^{i\delta} - e^{-i\delta}) c s \kappa_t \kappa_b \lambda^{12}$$

$$J_{Jkg} = \frac{-\text{Det.} C}{2F_c F_s} = \kappa_t \kappa_b \sin \delta \lambda^6$$

$$\frac{M_u}{m_t} = \begin{pmatrix} \lambda^7 u & 0 & 0 \\ 0 & \lambda^4 c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \frac{M_d}{m_b} = \begin{pmatrix} \lambda^4 d & 0 & 0 \\ 0 & \lambda^2 s & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

With our exact V(CKM), R=1 and R=L give

$R = 1,$

$$\tilde{M}^{(u)} = \begin{pmatrix} u\lambda^7, & 0, & 0 \\ -c\lambda^5, & c\lambda^4(1 + \frac{1}{6}\lambda^2), & c\kappa_t\lambda^6 \\ -\kappa_t e^{i\delta}\lambda^3(1 + \frac{1}{3}\lambda^2), & \kappa_t e^{i\delta}\lambda^2(1 - \frac{\lambda^2}{6} + [\kappa_b^2 - \frac{41}{360}]\lambda^4), & -e^{i\delta}(1 - \kappa_t \frac{\lambda^4}{2} - \kappa_t^2 \frac{\lambda^6}{3}) \end{pmatrix}$$

$$\tilde{M}^{(d)} = \begin{pmatrix} d\lambda^4(1 + \frac{2}{3}\lambda^2), & 0, & 0 \\ 0, & s\lambda^2(1 + \frac{\lambda^2}{3} + [\frac{8}{45} + \frac{\kappa_b^2}{2}]\lambda^4), & s\kappa_b e^{i\delta}\lambda^4(1 + \frac{2}{3}\lambda^2) \\ 0, & \kappa_b\lambda^2(1 + \frac{\lambda^2}{3} + [\frac{8}{45} + \kappa_b^2]\lambda^4), & -e^{i\delta}(1 - \kappa_b^2 \frac{\lambda^4}{2} - \kappa_b^2 \frac{\lambda^6}{3}) \end{pmatrix}.$$

$R = L,$

$$\tilde{M}^{(u)} = \begin{pmatrix} (c + \kappa_t^2\lambda)\lambda^6, & -(c + \kappa_t^2)\lambda^5, & \kappa_t\lambda^3(1 + \frac{1}{3}\lambda^2) \\ -(c + \kappa_t^2)\lambda^5, & c\lambda^4(1 - \frac{1}{3}\lambda^2), & -\kappa_t\lambda^2 + \frac{\kappa_t}{6}\lambda^4 + O(\lambda^6) \\ \kappa_t\lambda^3(1 + \frac{1}{3}\lambda^2), & -\kappa_t\lambda^2 + \frac{\kappa_t}{6}\lambda^4 + O(\lambda^6), & 1 - \kappa_t^2 \frac{\lambda^4}{2} - \kappa_t^2 \frac{\lambda^6}{3} \end{pmatrix}$$

$$\tilde{M}^{(d)} = \begin{pmatrix} d\lambda^4(1 + \frac{2}{3}\lambda^2), & 0, & 0 \\ 0, & s\lambda^2 + (\kappa_b + \frac{s}{3})\lambda^4 + (\frac{8}{45}s + \frac{2\kappa_b^2}{3})\lambda^6, & \kappa_b e^{i\delta}(-\lambda^2 + (s - \frac{1}{3})\lambda^4) + O(\lambda^6) \\ 0, & \kappa_b e^{-i\delta}(-\lambda^2 + [s - \frac{1}{3}]\lambda^4) + O(\lambda^6), & 1 - \kappa_b^2\lambda^4 + \kappa_b^2(s - \frac{2}{3})\lambda^6 \end{pmatrix}$$

Useful textures to find symmetries behind Yukawa couplings at the fundamental scale. BUT

For the  $R=L$  case, the form is symmetrical, and I could not find a form from the  $U(1)$  symmetry, employing the Froggatt-Nilsson mechanism.

For  $R=1$ , the mass matrix is not necessarily symmetrical, and there may be more freedom, employing the F-N mechanism. I have not tried since we know that the axion solution is consistent with the Kobayashi-Maskawa type weak CP violation. Of course, there are more freedom in choosing  $R$ , and at present we do not have a predictive scheme.



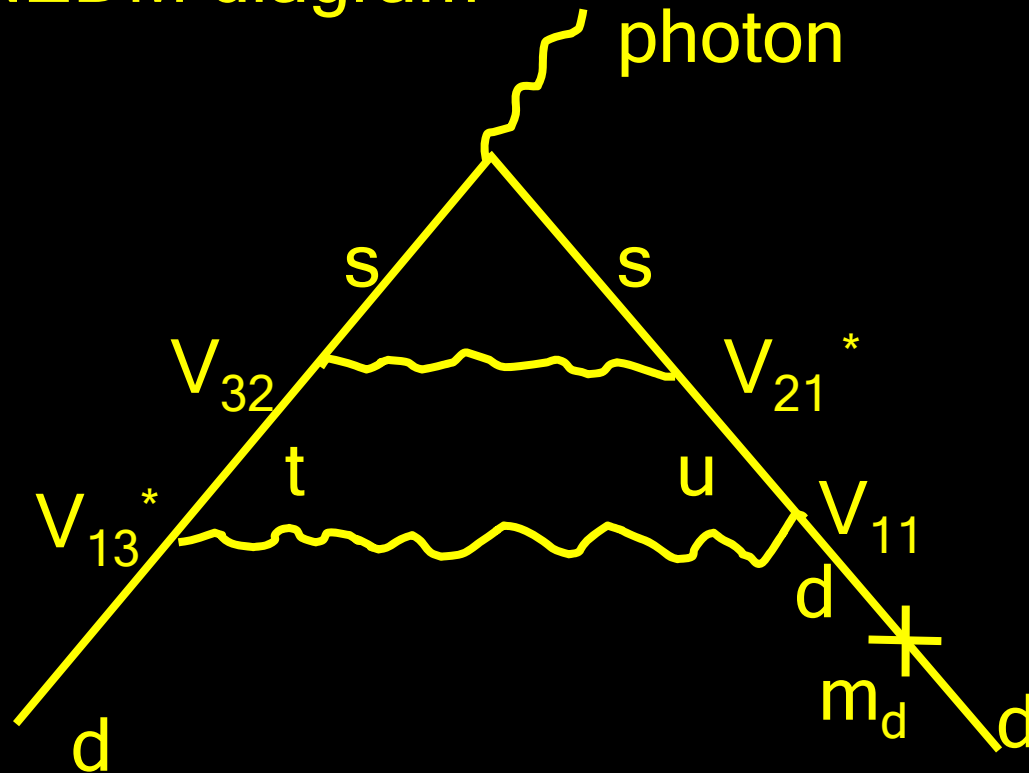
In any case, either  $\text{Det } V$  is real or there must be an axion. Both of them solves the strong CP problem.

Consider, for example, there is no axion and  $\text{Det } V$  is not real. But, there is no dangerous NEDM diagram. Since  $\text{Det } V$  is not real, try that it has a phase  $3\phi$

$$\left. \begin{aligned} V_{12}V_{23}V_{31} &= (s_1^2 s_2^2 c_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}) \\ -V_{12}V_{21}V_{33} &= (s_1^2 c_2^2 c_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}) \\ V_{13}V_{21}V_{32} &= (s_1^2 c_2^2 s_3^2 - c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}) \\ -V_{13}V_{22}V_{31} &= (s_1^2 s_2^2 s_3^2 + c_1 c_2 c_3 s_1^2 s_2 s_3 e^{i\delta}) \end{aligned} \right\} e^{i3\phi}$$

Can  $\phi$  appear as a physical one? Seems not.

Consider the NEDM diagram



$$V_{13}^* V_{32} V_{21}^*$$

appear in the determinant  $t$  with phase  $e^{3i\Phi}$ .

The (11) element makes it real, so the overall phase does not appear. But  $\Phi = 0$  is simple enough to see the essence.

### 3. The $\mu$ problem

The good choice of the phases such that  $\text{Det. } V(\text{CKM}) = \text{real}$  is related to the PQ symmetry.

The PQ symmetry needs two Higgs doublets or heavy quarks. SUSY, probably most of us here study, needs two Higgs doublets.

With two Higgs doublets,  $H_u$  and  $H_d$ , the PQ does not like to write the following term in  $W$ ,

**Tree  $W$ , no  $H_u H_d$**

K-Nilles (1984)

This is a serious problem in the MSSM. Better to break the  $SU(2) \times U(1)$  at the electroweak scale.

The  $\mu$  problem can be stated in several disguises:

1. The doublet-triplet splitting problem in SUSY GUTs,
2. Is there PQ symmetry?
3. How large is the  $\mu$  term?
4. The  $B_\mu$  problem in the GMSB.
5. Why only 1 pair of Higgs doublets? ←

To forbid at the GUT scale, PQ or R symmetries are used.

$$W = \mu H_u H_d, \quad \text{forbidden}$$

$$\text{if } X(H_u) = 1, X(H_d) = 1$$



To generate a TeV scale  $\mu$  is another problem. There are some ways such as,

Nonrenormalizable superpotential helps,

Kim-Nilles (in  $W$ )

SUSY breaking scale is used

Giudice-Masiro (Kaehler potential)

In any case, a symmetry in particular the PQ symmetry might be behind this story.

Since the PQ symmetry is good, one can use this

$$W = \frac{S_1 S_2}{M_P} H_u H_d$$

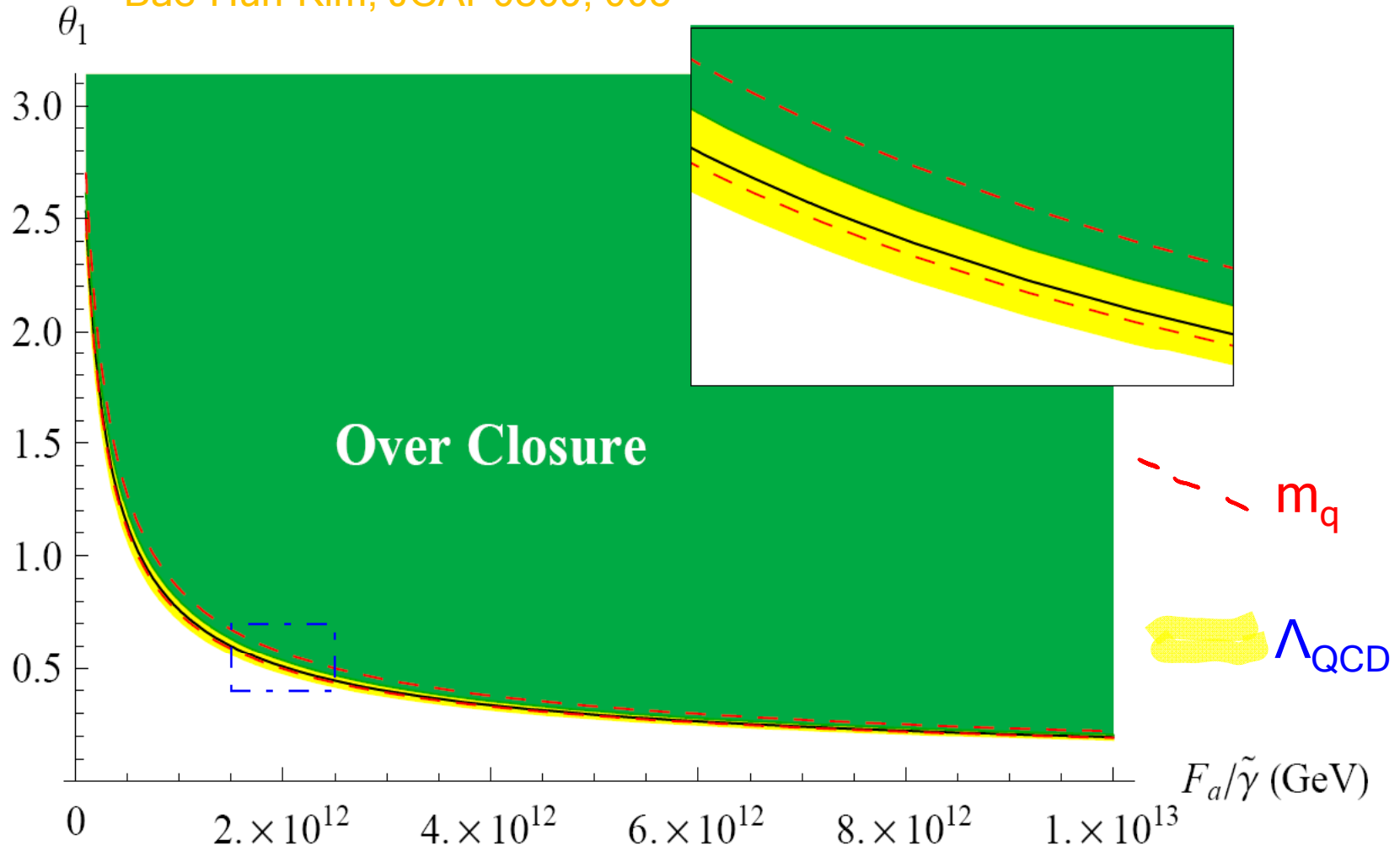
if  $X(S_1) = -1, X(S_2) = -1, X(H_u) = 1, X(H_d) = 1$   
 $\langle S_1 \rangle = \langle S_2 \rangle \approx 10^{10-12} \text{ GeV}$

Also, an axion solution of the strong CP problem

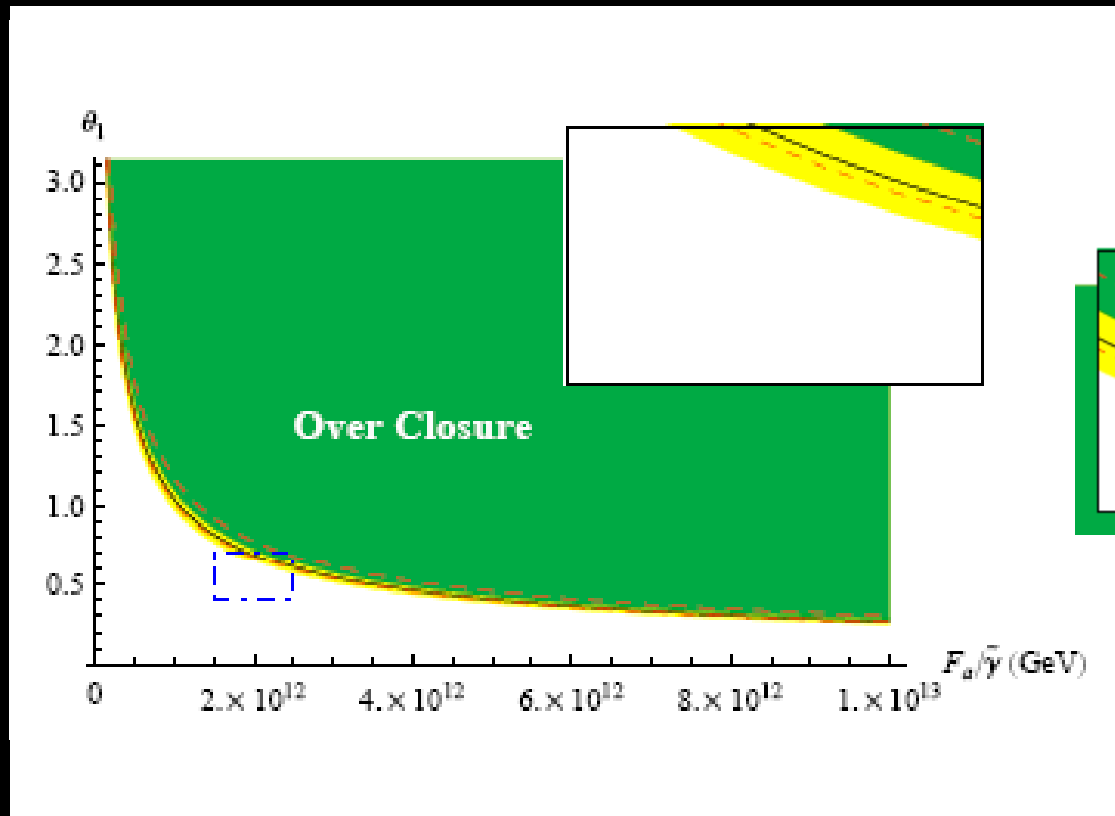
This leads to the very light axion and axion cosmology.

The axion cosmology restricts the axion decay constant below  $10^{12}$  GeV, but it is in fact related to the initial misalignment angle  $\theta_1$ .

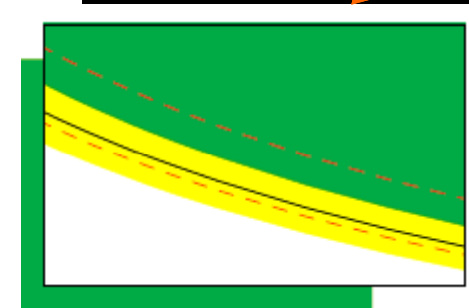
Bae-Huh-Kim, JCAP0809, 005



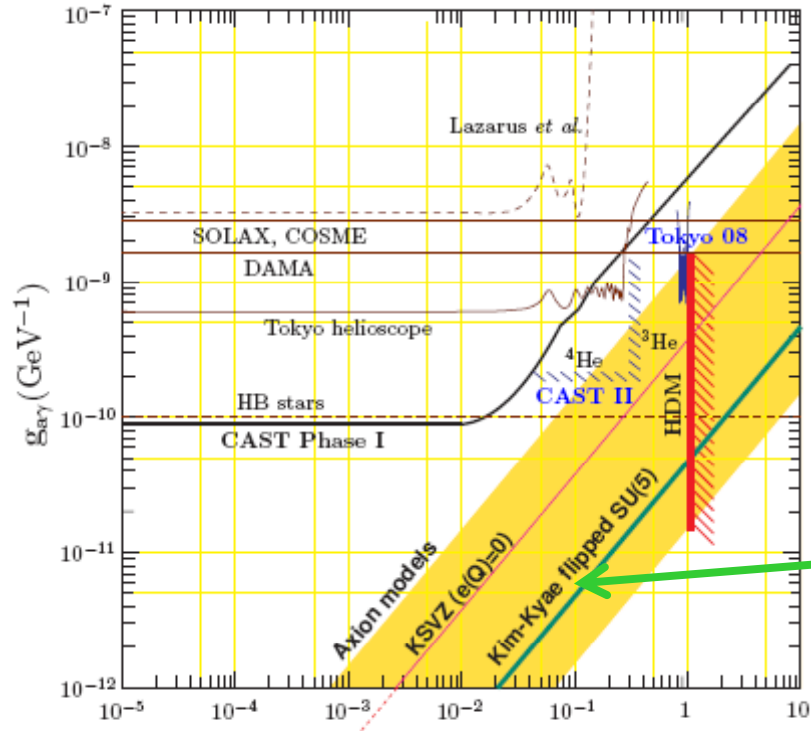
If we do not take into account the overshoot factor and the anharmonic correction,



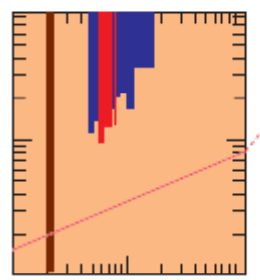
Inclusion of these showed the region, prev. figure



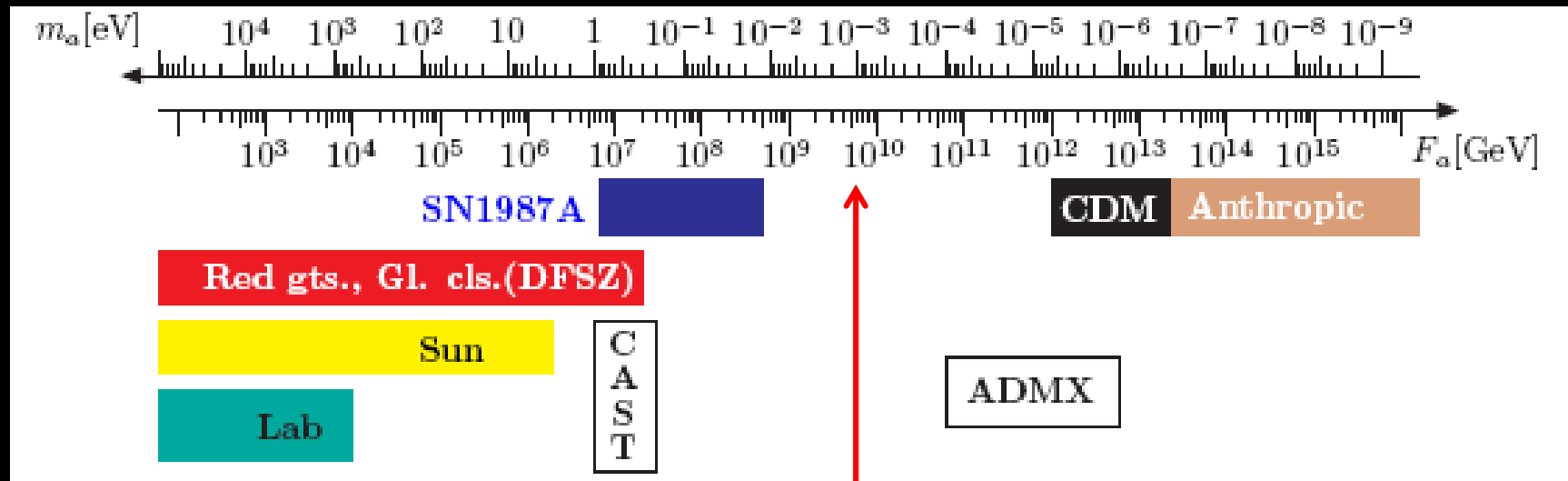




It depends on models.  
There are not many calculations in string models.



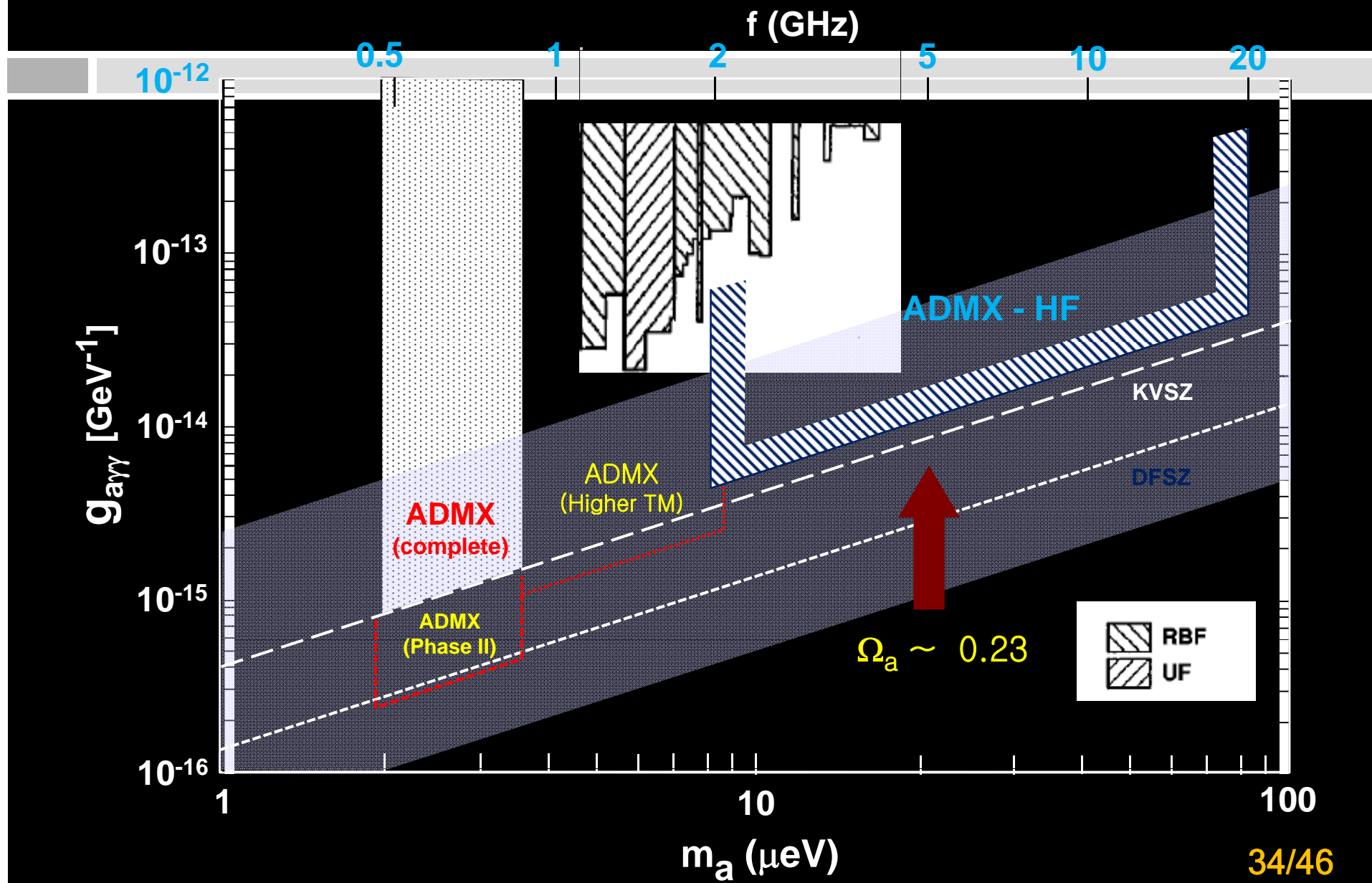
KSVZ		DFSZ	
$Q_{em}$	$c_{a\gamma\gamma}$	$x = \tan \beta = v_u/v_d$ ,	same Higgs for $(q^c, e)$ masses, $c_{a\gamma\gamma}$
0	-1.95	any $x$ ,	$(d^c, e)$ 0.72
$\pm \frac{1}{3}$	-1.28	any $x$ ,	$(u^c, e)$ -1.28
$\pm \frac{2}{3}$	0.72		
$\pm 1$	4.05		
$(m, m)$	-0.28		



White dwarf bound  
 (1<sup>st</sup> hint at the center of the axion window)

# ADMX Phase II & ADMX-HF Coverage

van Bibber at ASK 2011



## Orbifold compactification of heterotic string:

Dixon-Harvey-Vafa-Witten (1986)

Ibanez-Kim-Nilles-Quevedo (1987) on SM

1. We used the orbifold idea toward the composite invisible axion solving  $\mu$ :

Chun, Kim, Nilles, NPB 370, 105 (1992)

2. Approximate symmetry from orbifold compactification was used to obtain a PQ symmetry:

K.-S. Choi, I.-W. Kim, Kim, JHEP 0703, 116 (2007)

[hep-ph/0612107].

3. Approximate R-symmetry from orbifold compactification was used to obtain a power-law generated  $\mu$ :

R. Kappl, H. P. Nilles, S. Ramos-Sanches, M. Ratz,  
K. Schmidt-Hoberg, P. K.S. Vaudrevange,

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# One pair of Higgs doublets with $SU(3)_W$

Z(12-I) orbifold: [JEK, plb 656, 207 (2007) [arXiv:0707.3292]

The shift vector and Wilson line is taken as

$$V = (1/12)(6 \ 6 \ 6 \ 2 \ 2 \ 2 \ 3 \ 3)(3 \ 3 \ 3 \ 3 \ 3 \ 1 \ 1 \ 1)'$$

$$a_3 = (1/12)(\underline{1 \ 1 \ 2} \ \underline{0 \ 0 \ 0 \ 0 \ 0})(\underline{0 \ 0 \ 0 \ 0 \ 0} \ 1 \ 1 \ -2)'$$

Gauge group is

$$SU(3)_c \times SU(3)_W \times SU(5)' \times SU(3)' \times U(1)s$$

Lee-Weinberg electroweak model and **no exotics**



## The SM spectrum.

$P + [kV + ka]$	No. $\times$ (Repts.) $_Y [Q_1, Q_2, Q_3, Q_4, Q_5]$	$\Gamma$	Label
$\left(\frac{-1}{3} \frac{-1}{3} \frac{-2}{3} \frac{2}{3} \frac{-1}{3} \frac{-1}{3} 0 0\right) (0^8)'_{T_{4-}}$	$3 \cdot (\mathbf{3}, \mathbf{2})_{1/6}^L [0,0,0;0,0]$	1	$q_1, q_2, q_3$
$\left(\frac{1}{6} \frac{1}{6} \frac{5}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{2} \frac{1}{2}\right) (0^8)'_{T_{4-}}$	$2 \cdot (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}^L [-3,3,2;0,0]$	3	$u^c, c^c$
$\left(\frac{-1}{3} \frac{-1}{3} \frac{-2}{3} \frac{1}{3} \frac{1}{3} \frac{1}{3} \frac{-1}{4} \frac{-1}{4}\right) \left(\frac{1^5}{4} \frac{1}{12} \frac{1}{12} \frac{1}{12}\right)'_{T_{7+}}$	$(\bar{\mathbf{3}}, \mathbf{1})_{-2/3}^L [0,6,-1;5,1]$	1	$t^c$
$\left(\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{-1}{6} \frac{-1}{6} \frac{-1}{6} 0 0\right) (0^5 \frac{-1}{3} \frac{-1}{3} \frac{-1}{3})'_{T_{2_0}}$	$(\bar{\mathbf{3}}, \mathbf{1})_{1/3}^L [3,-3,0;0,-4]$	-1	$d^c$
$\left(\frac{1}{6} \frac{1}{6} \frac{5}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{-1}{2} \frac{-1}{2}\right) (0^8)'_{T_{4-}}$	$2 \cdot (\bar{\mathbf{3}}, \mathbf{1})_{1/3}^L [-3,3,-2;0,0]$	1	$s^c, b^c$
$\left(\frac{-1}{3} \frac{-1}{3} \frac{1}{3} \frac{2}{3} \frac{-1}{3} \frac{2}{3} 0 0\right) (0^8)'_{T_{4-}}$	$(\mathbf{1}, \mathbf{2})_{-1/2}^L [-6,6,0;0,0]$	1	$l_1, l_2, l_3$
$(0 0 0 \frac{2}{3} \frac{-1}{3} \frac{2}{3} \frac{-1}{4} \frac{-1}{4}) \left(\frac{1^5}{4} \frac{1}{12} \frac{1}{12} \frac{1}{12}\right)'_{T_{1_0}}$	$(\mathbf{1}, \mathbf{2})_{1/2}^L [0,6,-1;5,1]$	0	$H_u$
$\left(\frac{-1}{3} \frac{-1}{3} \frac{1}{3} \frac{1}{3} \frac{-2}{3} \frac{1}{3} \frac{-1}{4} \frac{-1}{4}\right) \left(\frac{1^5}{4} \frac{1}{12} \frac{1}{12} \frac{1}{12}\right)'_{T_{7+}}$	$(\mathbf{1}, \mathbf{2})_{-1/2}^L [-6,0,-1;5,1]$	-2	$H_d$

Note that  $U(1)_F$  charges of SM fermions are odd and Higgs doublets are even. By breaking by VEVs of even  $\Gamma$  singlets, we break  $U(1)_F$  to a discrete matter parity  $P$  or Dreiner's matter parity  $Z_6$  is realized; dim. 5 operator  $qqql$  [Sakai-Yanagida, Hall-Weinberg] is not allowed.

After removing vectorlike representations by  $\Gamma =$  even integer singlets, the starred representations remain

$P + n[V \pm a]$	$\Gamma$	No. $\times$ (Repts.) $\gamma[\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_4, \mathcal{Q}_5]$
$(\frac{1}{6} \frac{1}{6} \frac{-1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6} \frac{1}{6}) (\frac{-3}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{-1}{4} \frac{-1}{4} \frac{-1}{4})'_{T1_-}$	2	$(\mathbf{1}; \bar{\mathbf{5}}', \mathbf{1})^L_{0[3,3,1;1,-1]}$
$(\frac{1}{6} \frac{1}{6} \frac{-1}{6} \frac{-1}{6} \frac{-1}{6} \frac{-1}{6} \frac{0}{6}) (\frac{1}{2} \frac{1}{2} \frac{-1}{2} \frac{-1}{2} \frac{-1}{2} \frac{-1}{2} \frac{-1}{2})'_{T2_+}$	-1	$\star(\mathbf{1}; \mathbf{10}', \mathbf{1})^L_{0[3,-3,0;-2,-2]}$
$(0^6 \frac{1}{4} \frac{-3}{4}) (\frac{3}{4} \frac{-1}{4} \frac{-1}{4} \frac{-1}{4} \frac{-1}{4} \frac{1}{4} \frac{1}{4})'_{T3}$	-1	$(2_n; \mathbf{5}', \mathbf{1})^L_{0[0,0,-1;-1,3]}$
$(0^6 \frac{3}{4} \frac{-1}{4}) (\frac{-3}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{-1}{4} \frac{-1}{4} \frac{-1}{4})'_{T9}$	1	$(2_n; \bar{\mathbf{5}}', \mathbf{1})^L_{0[0,0,1;1,-3]}$
$(0^3 \frac{-1}{3} \frac{-1}{3} \frac{-1}{3} \frac{1}{4} \frac{1}{4}) (\frac{-3}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4})'_{T7_0}$	-1	$\star(\mathbf{1}; \bar{\mathbf{5}}', \mathbf{1})^L_{0[0,-6,1;1,1]}$
$(\frac{1}{6} \frac{1}{6} \frac{-1}{6} \frac{1}{6} \frac{1}{6} \frac{-1}{6} \frac{-1}{6}) (\frac{3}{4} \frac{-1}{4} \frac{-1}{4} \frac{-1}{4} \frac{-1}{4} \frac{1}{4} \frac{1}{4})'_{T7_-}$	0	$(\mathbf{1}; \mathbf{5}', \mathbf{1})^L_{0[3,3,-1;-1,3]}$
$(0^6 \frac{-1}{2} \frac{-1}{2}) (-10000000)'_{T6}$	-2	$3 \cdot (\mathbf{1}; \bar{\mathbf{5}}', \mathbf{1})^L_{0[0,0,-2;-4,0]}$
$(0^6 \frac{-1}{2} \frac{-1}{2}) (10000000)'_{T6}$	-2	$2 \cdot (\mathbf{1}; \mathbf{5}', \mathbf{1})^L_{1[0,0,-2;4,0]}$
$(0^6 \frac{1}{2} \frac{1}{2}) (-10000000)'_{T6}$	2	$2 \cdot (\mathbf{1}; \bar{\mathbf{5}}', \mathbf{1})^L_{-1[0,0,2;-4,0]}$
$(0^6 \frac{1}{2} \frac{1}{2}) (10000000)'_{T6}$	2	$3 \cdot (\mathbf{1}; \mathbf{5}, \mathbf{1})^L_{0[0,0,2;4,0]}$

The hidden  $SU(5)'$  spectrum.

Note that  $10'+5^*$  remain.  
It leads to a dynamical SUSY breaking.

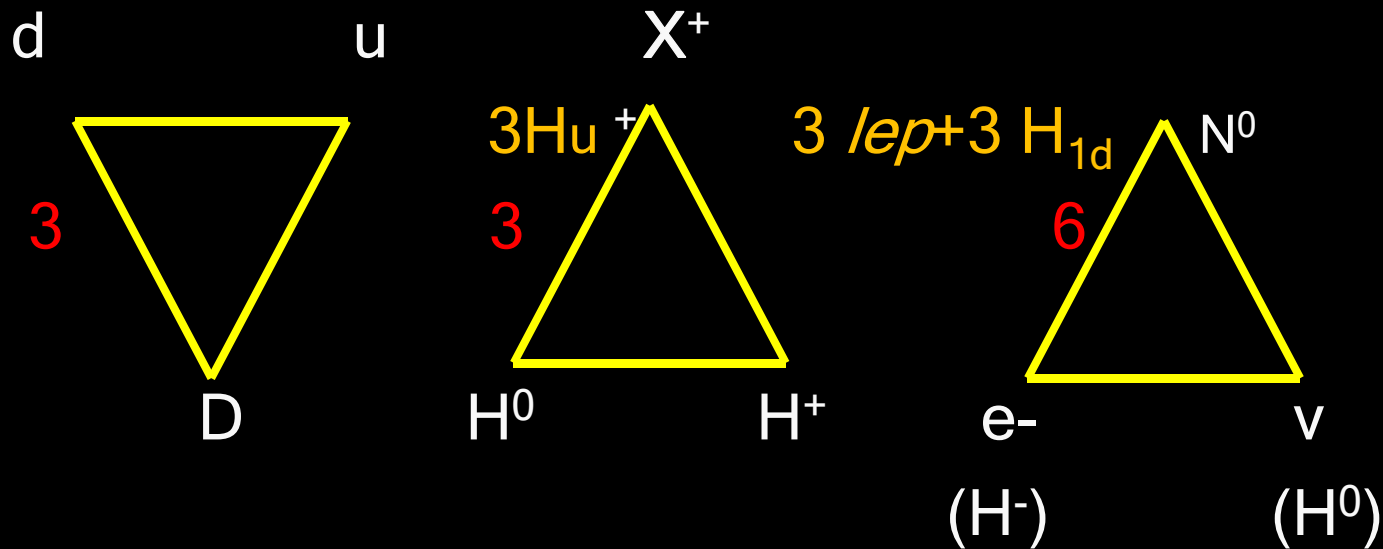
$$SU(3)_c \times SU(3)_W \times SU(2)_N \\ \times SU(5)' \times SU(3)'$$

## Three quark families appear as

$$3 (3_c, 3_w)$$

At low energy, we must have **nine**  $3_w^*$  to cancel  $SU(3)_w$  anomaly.

Both  $H_u$  and  $H_d$  appear from  $3^*$ . It is in contrast to the other cases such as in  $SU(5)$  or  $SO(10)$ . Now, the  $H_u$  and  $H_d$  coupling must come from  $3_w^* 3_w^* 3_w^*$  coupling.



There remain three pairs of  $3_w^*(H^+)$  and  $3_w^*(H^-)$  plus three families of  $3_w$ (quark) and  $3_w^*$ (lepton)



Thus, there appears the Levi-Civita symbol and two epsilons are appearing, in  $SU(3)_W$  space,  $a, b, c$  and in flavor space,  $I, J, \dots$ .  
Therefore, in the flavor space the  $H_u-H_d$  mass matrix is antisymmetric and hence its determinant is zero.

It is interesting to compare an old QCD idea and the Kim's  $SU(3)$  model:

Introduction of color:

56 of old  $SU(6)$  in 1960s = completely symm:  $\Omega^- = s^\uparrow s^\uparrow s^\uparrow$

But spin-half quarks are fermions  $\rightarrow$

introduce antisymmetric index =  $SU(3)$  color [Han-Nambu]

Introduction of flavor in the Higgs sector:

Lee-Weinberg  $SU(3)$ -weak gives

$3^*-3^*-3^*$   $SU(3)$ -weak singlet = antisymmetric gives  
antisymmetric bosonic flavor symmetry (SUSY)!

and one pair of Higgs doublets is massless.

We had this in the orbifold compactification.  
I never thought of it as a GUT model.

With F-theory, we can talk about GUTs.



GUT gauge group :  $SU(6)_{\text{GUT}}$

Flavor unification: JEK, PLB107 (1982) 69

$$\mathbf{15}_L = \begin{pmatrix} 0 & u^c & -u^c & u & d & D \\ -u^c & 0 & u^c & u & d & D \\ u^c & -u^c & 0 & u & d & D \\ -u & -u & -u & 0 & e^c & H_u^+ \\ -d & -d & -d & -e^c & 0 & H_u^0 \\ -D & -D & -D & -H_u^+ & -H_u^0 & 0 \end{pmatrix},$$

$$\bar{\mathbf{6}}_L = \begin{pmatrix} d^c \\ d^c \\ d^c \\ N \\ \nu_e \\ -e \end{pmatrix}, \quad \bar{\mathbf{6}}'_L = \begin{pmatrix} D^c \\ D^c \\ D^c \\ N' \\ H_d^0 \\ -H_d^- \end{pmatrix}.$$

This GUT contains the previous  $SU(3)_W$ . So, if we succeed in unification with  $SU(3)_c$ , then the needed flavor symmetry will result. In F-theory, we succeeded.

In F-theory, we succeeded in obtaining the  $SU(3) \times SU(3) \times U(1)$  phenomenology. One pair problem of the Higgs doublets is realized in an ultraviolet completed theory.  
In addition, we could show that if  $Z'$  is present, it is much heavier above the electroweak scale.



The adjoint 248 of E8 branches under SU(6)xSU(2)xSU(3) as

$$248 \rightarrow (35, 1, 1) + (1, 3, 1) + (1, 1, 8) + (20, 2, 1) \\ + (15, 1, 3) + (6^*, 2, 3) + \text{c.c.} \quad [\text{Corrected PS table}]$$

Adjoint are

$$\begin{aligned} \text{SU}(6) : & (\underline{1 \bar{1} 0 0 0 0 0 0}); \\ \text{SU}(2) : & T_{\pm} = (0 0 0 0 0 0 \underline{1 \bar{1}}); \\ \text{SU}(3)_{\perp} : & I_{\pm} = \pm(0 0 0 0 0 0 \underline{1 1}), \\ & V_+ = (-^6 ++), V_- = (+^6 --), \\ & U_+ = (-^6 --), U_- = (+^6 ++), \end{aligned}$$

We represented in terms of physicists' matrices.

Matters are

$$\begin{aligned} (15, 1, 3) &= \left\{ \frac{(+ + - - - - - -), (+ + - - - - + +)}{(\underline{1 1 0 0 0 0 0 0})} \right. \\ (\bar{6}, 2, 3) &= \left\{ \frac{(\bar{1} 0 0 0 0 0 \bar{1} 0), (\bar{1} 0 0 0 0 0 1 0)}{(\underline{+ + + + + - + -})} \right. \\ (20, 2, 1) &= (\underline{+ + + - - - + -}). \end{aligned}$$

## SU(6) x SU(2) x SU(3):

$SU(6)$  : diag. generator :  $Y_6$

$SU(2)$  : diag. generator :  $X_3$

$SU(3)$  : diag. generators :  $F_3, F_8$

$$F_3 = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1)$$

$$F_8 = (-1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad 0 \quad 0)$$

$$X_3 = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1 \quad 1)$$

$$\begin{aligned} X &= -F_8 + X_3 \\ &= (1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1) \end{aligned}$$

$\Lambda_8$  touches the 6<sup>th</sup> component of SU(6). The same SU(6) representation contains two values of X. But SU(5) rep. has the same X.



# Conclusion

Here, I talked two topics beyond the SM paying attention to my recent papers.

1. There is no  $Z'$  below 10 TeV, otherwise our wisdom to the standard model is in trouble.
2. A useful suggestion for the CKM matrix.
3. One pair of Higgs doublets in the MSSM from string: the  $\mu$  problem.

