7th Patras Workshop on Axions, WIMPS and WISPs, Mykonos, June 26 - July 1, 2011

The Chiral Magnetic Effect and the Axions

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BROOKHAVEN NATIONAL LABORATORY

- This talk will not rely on the existence of axions; however the axion <u>concept</u> will appear very useful in formulating the theory of anomaly-induced phenomena in QCD

- Moreover, if the axions do exist, the phenomena discussed in the talk will be widespread in the Universe, at large scales

Outline

- Introduction:
  i) axial anomaly and geometry of gauge theories;
  ii) AdS/CFT correspondence, axions and sphalerons;
  iii) anomalies and relativistic hydrodynamics

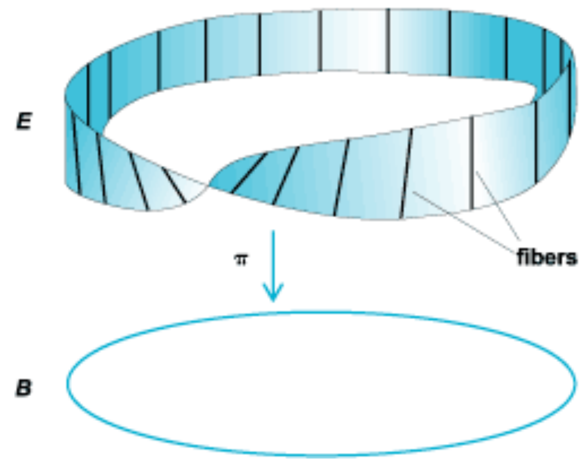
- The Chiral Magnetic Effect and axions

- The Chiral MagnetoHydroDynamics (CMHD) :
  relativistic hydrodynamics with axial anomaly

- Evidence for CME at RHIC and LHC; future tests

Geometry and gauge theory

Geometry		Physics
Riemannian connection	⟷	Gauge field
Curvature tensor	⟷	Field strength tensor



Möbius strip, the simplest nontrivial example of a fiber bundle

Gauge theories "live" in a fiber bundle space that possesses non-trivial topology (knots, links, twists,...)

Chern-Simons forms

6. Applications to 3-manifolds

In this section M will denote a compact, oriented, Riemannian 3-manifold, and $F(M) \xrightarrow{\pi} M$ will denote its $SO(3)$ oriented frame bundle equipped with the Riemannian connection θ and curvature tensor Ω. For A, B skew symmetric matrices, the specific formula for P_1 shows $P_1(A \otimes B) = -(1/8\pi^2)\operatorname{tr} AB$. Calculating from (3.5) shows

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\} \,. \tag{6.1}$$

What does it mean for a gauge theory?

Chern-Simons theory

CHARACTERISTIC FORMS

$$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\} . \tag{6.1}$$

What does it mean for a gauge theory?

Geometry ⟷ Physics

Riemannian connection ⟷ Gauge field

Curvature tensor ⟷ Field strength tensor

$$S_{CS} = \frac{k}{8\pi} \int_M d^3x \; \epsilon^{ijk} \left(A_i F_{jk} + \frac{2}{3} A_i [A_j, A_k] \right)$$

Abelian

non- Abelian

Chern-Simons theory

$$S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)$$

Remarkable novel properties:

- gauge invariant, up to a boundary term
- topological - does not depend on the metric, knows only about the topology of space-time **M**
- when added to Maxwell action, induces a mass for the gauge boson - different from the Higgs mechanism!
- **breaks Parity invariance**

Chern-Simons theory and the vacuum of Quantum Chromodynamics

Equation:

$$D^\mu F^a_{\mu\nu} = 0$$

Belavin, Polyakov, Tyupkin, Schwartz

Solution:

$$A^a_\mu(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2+\rho^2}.$$

Coupling of space-time and color:

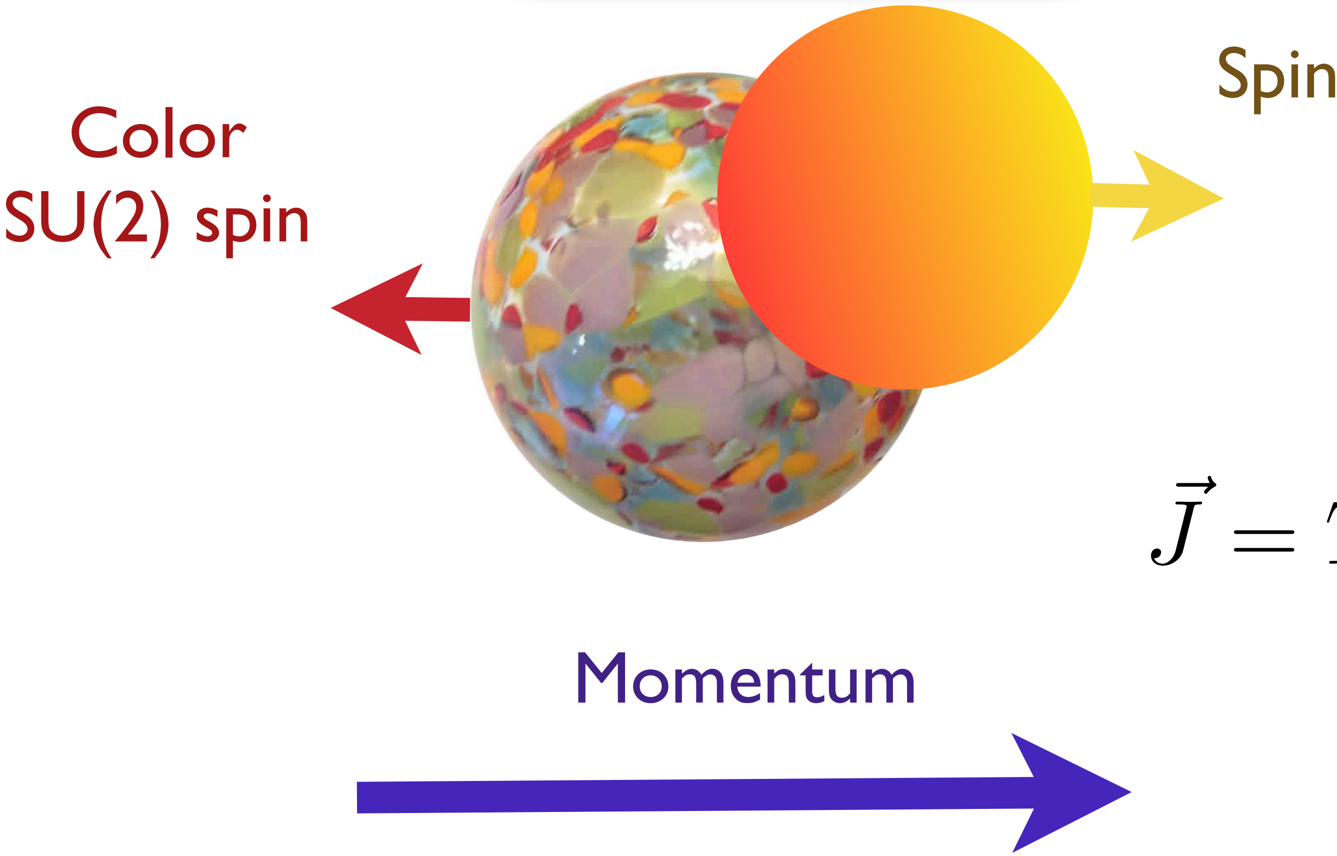
$$\eta_{a\mu\nu} = \begin{cases} \epsilon_{a\mu\nu} & \mu,\nu = 1,2,3, \\ \delta_{a\mu} & \nu = 4, \\ -\delta_{a\nu} & \mu = 4. \end{cases}$$

Integer $\quad Q = \int d\sigma_\mu K_\mu$

$$K_\mu = \frac{1}{16\pi^2}\epsilon_{\mu\alpha\beta\gamma}\left(A^a_\alpha \partial_\beta A^a_\gamma + \frac{1}{3} f^{abc} A^a_\alpha A^b_\beta A^c_\gamma\right)$$ Chern-Simons current

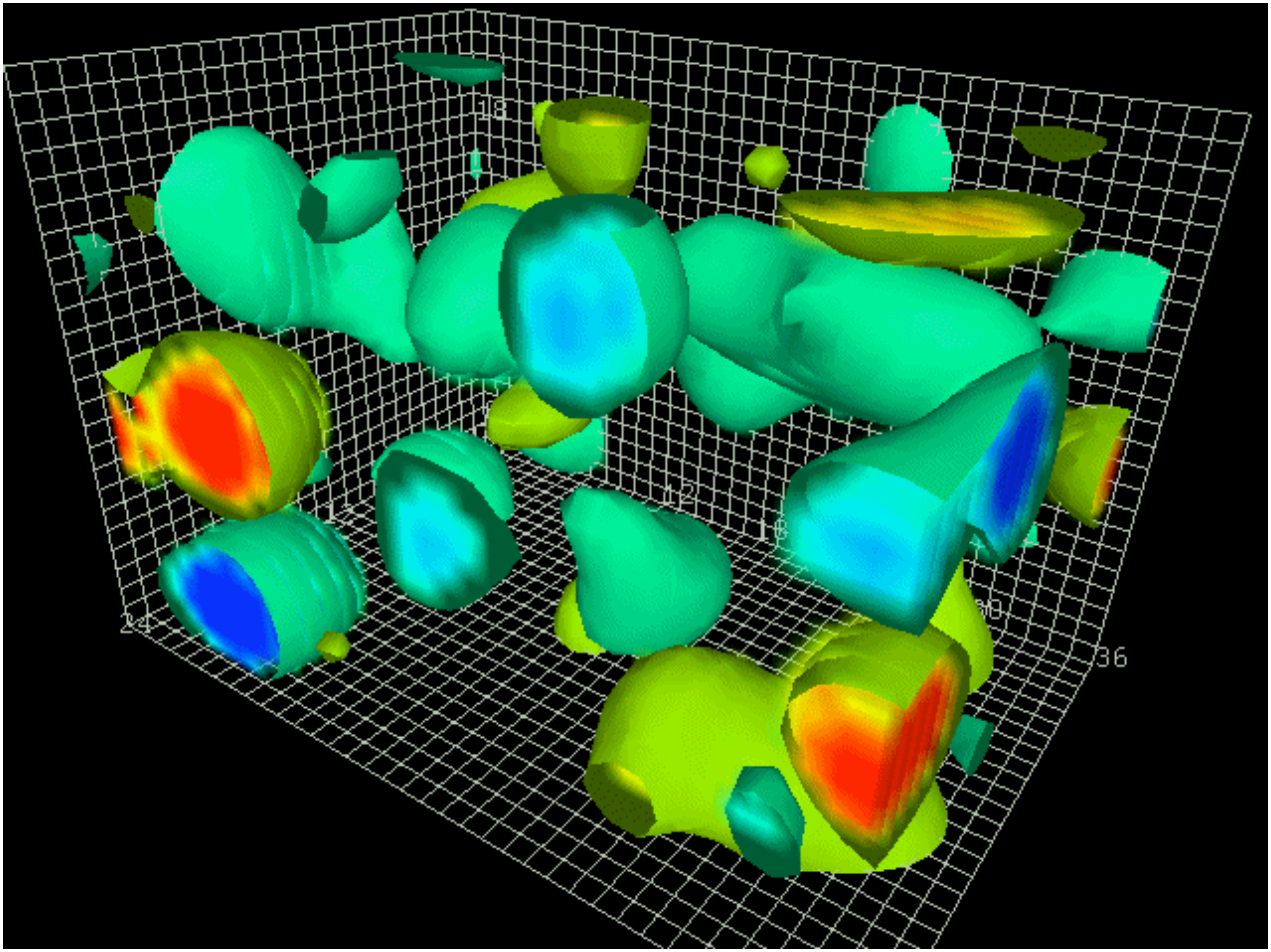
Topology-induced change of chirality

Right $\leftrightarrow$ Left



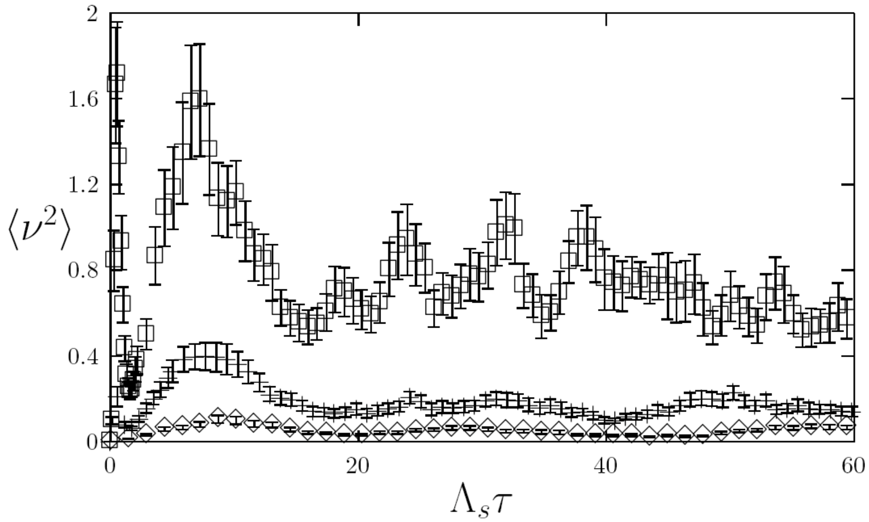
$$\vec{J} = \vec{T} + \vec{S}$$

Topological number fluctuations in QCD vacuum

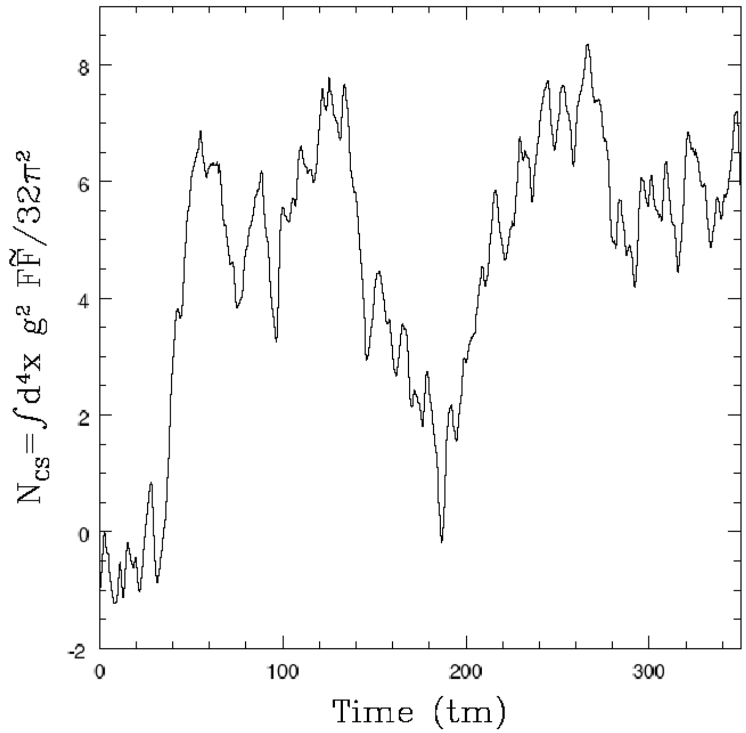


D. Leinweber

Topological transitions in QCD are seen in real-time lattice simulations



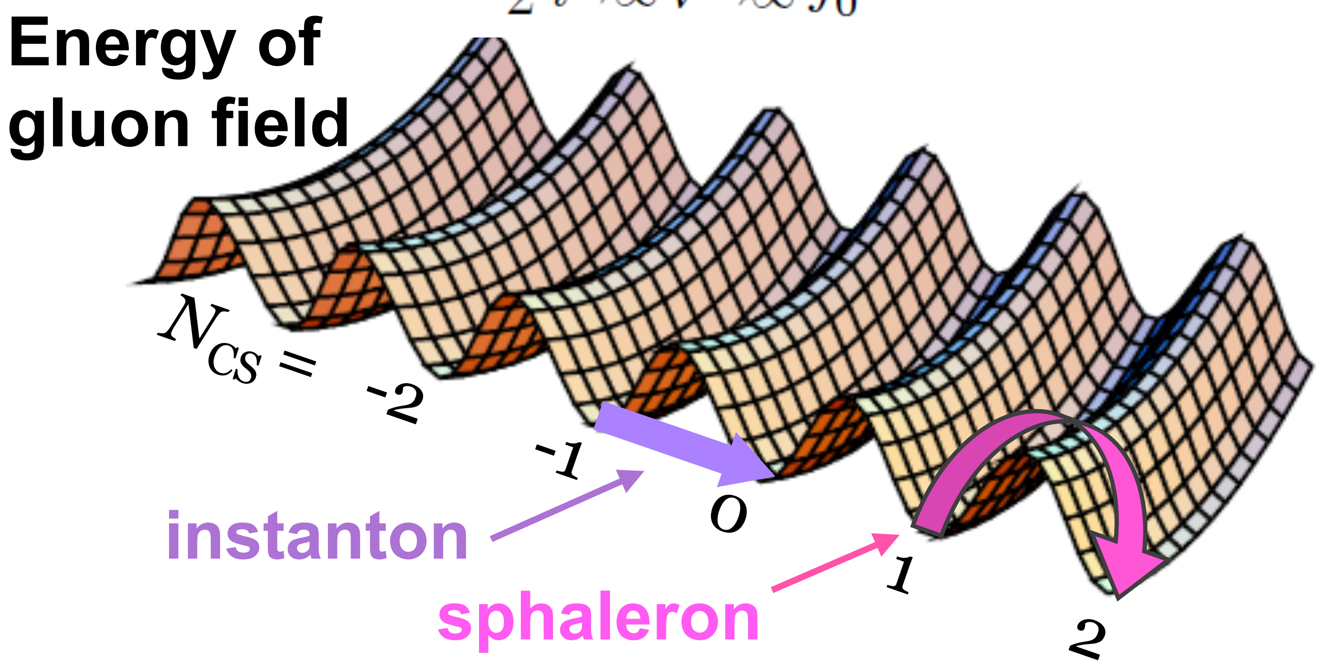
DK, A.Krasnitz and R.Venugopalan,
Phys.Lett.B545:298-306,2002



P.Arnold and G.Moore,
Phys.Rev.D73:025006,2006

Sphaleron transitions
at finite energy or temperature

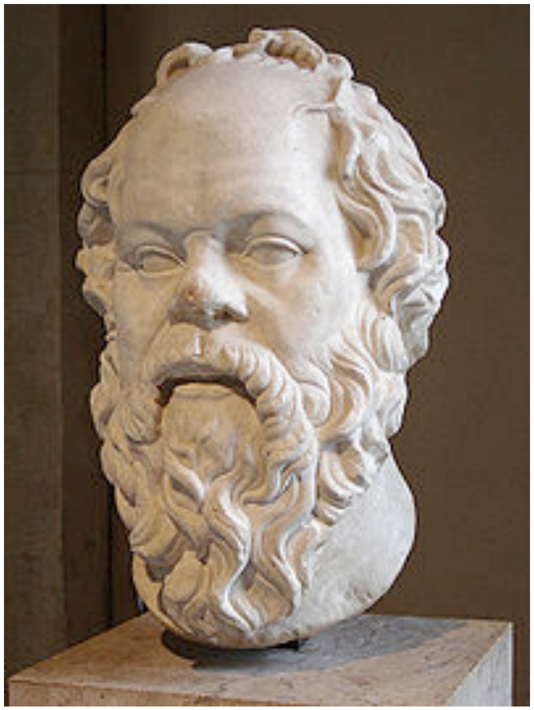
$$\Gamma = \frac{1}{2} \lim_{t\to\infty} \lim_{V\to\infty} \int_0^t \langle (q(x)q(0) + q(0)q(x)) \rangle d^4x$$



Sphalerons:
random walk of
topological charge at finite T: $\langle Q^2 \rangle = 2\Gamma V t, \quad t \to \infty$

Is this necessarily a classical (= weak coupling) phenomenon?

The metaphor of the cave, 380 B.C.



Socrates (Σωκράτης)
469 - 399 B.C.

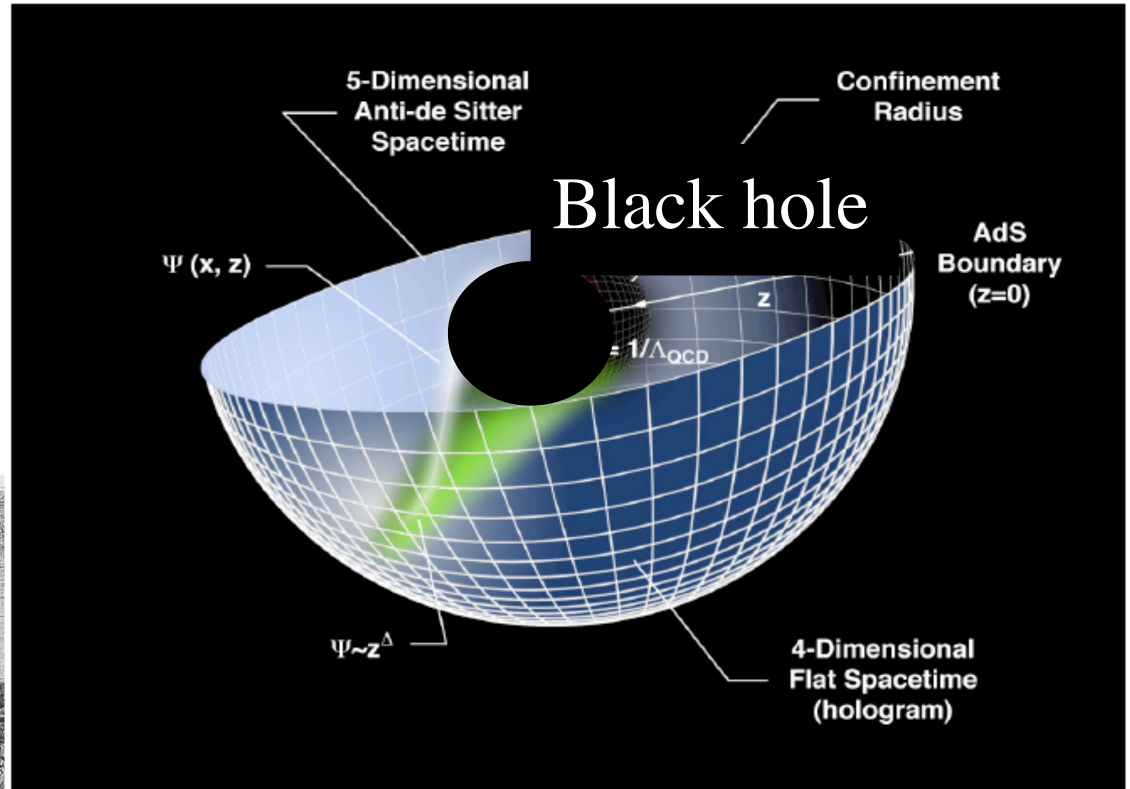
“Physical objects and physical events are only "shadows" of their ideal or perfect forms, and exist only to the extent that they instantiate the perfect versions of themselves”

Socrates, in Plato’s “Republic”

“The prisoners would take the shadows to be real things and the echoes to be real sounds, not just reflections of reality, since they are all they had ever seen or heard.”

The metaphor of the cave, 2011 A.D.:

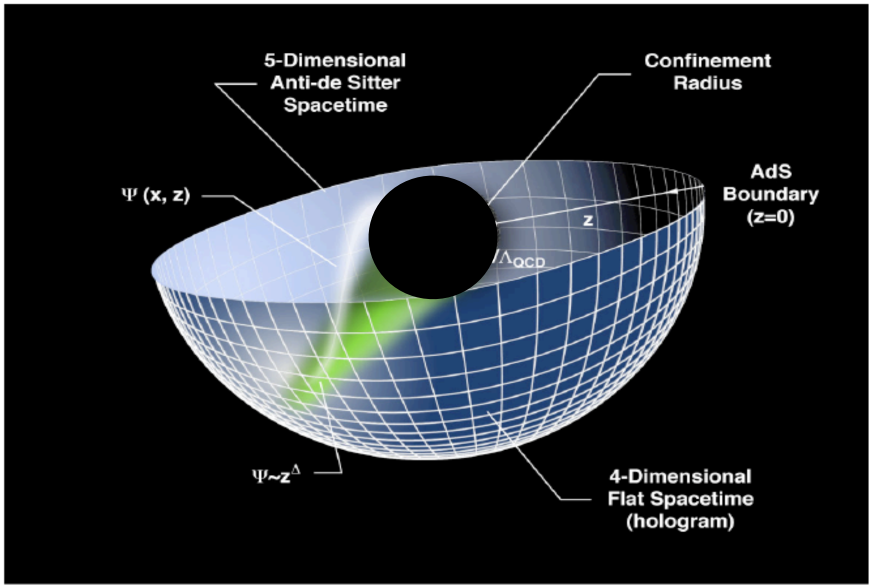
AdS/CFT correspondence



What is the low-energy theory of matter at strong coupling?

“The prisoners would take the shadows to be real things and the echoes to be real sounds, not just reflections of reality, since they are all they had ever seen or heard.”

Effective theory: hydrodynamics



Holographic view:

Particle contents of supergravity:
gravitons, dilatons, axions
= fields on the boundary

Caveman's view:

- Shear viscosity
- Bulk viscosity

Deviation from conformal symmetry

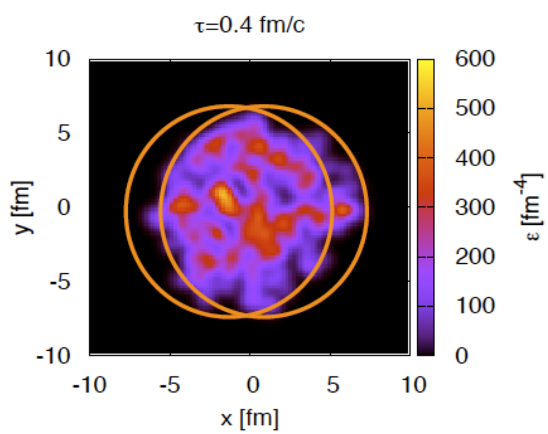
- Rate of topological transitions

AdS5 "Reality":

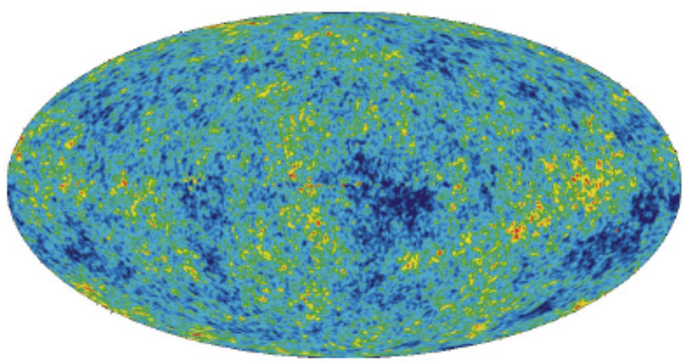
- Graviton propagation
- Dilaton propagation
- **Axion propagation**

Hydrodynamics: an effective low-energy Theory **O**f **E**verything (**TOE**)

- Hydrodynamics states that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)



Little Bang
(heavy ion collision)



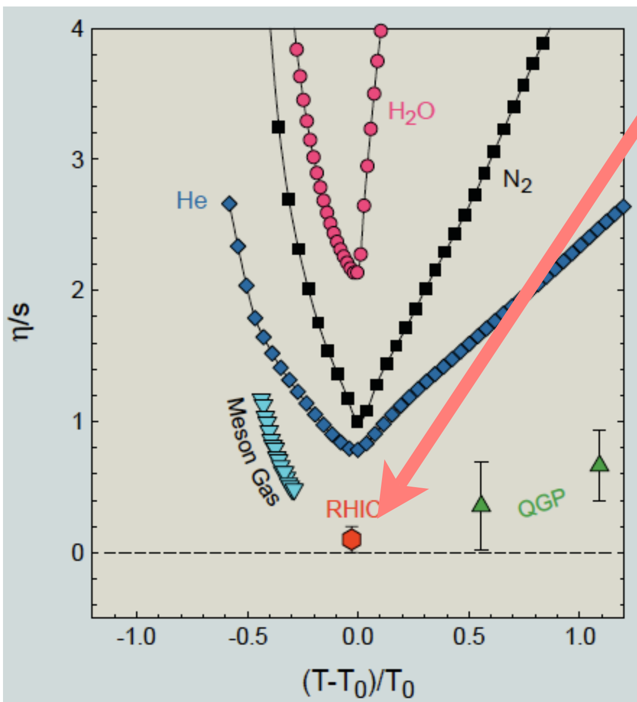
WMAP, Astrophys.J.Suppl.170:288,2007

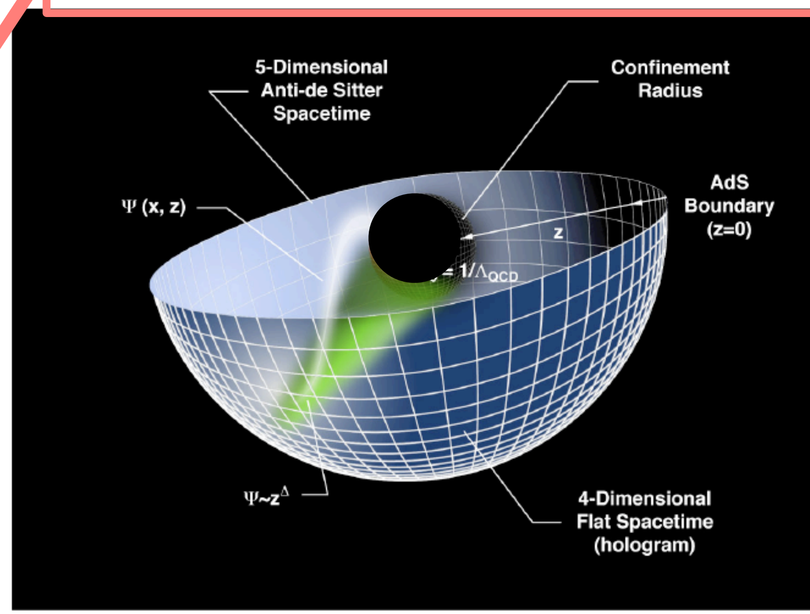
Big Bang

Quantifying the transport properties of QCD matter

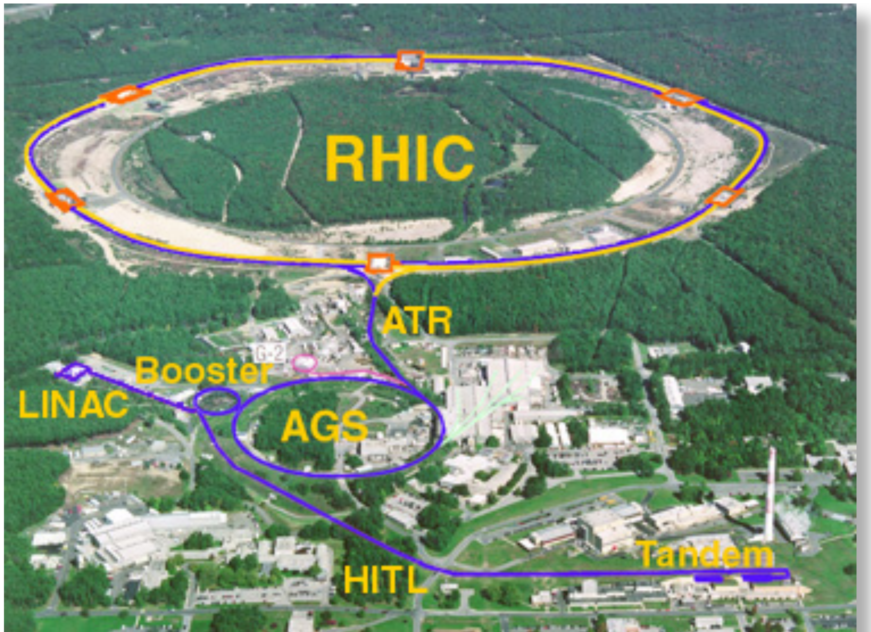
- Hydrodynamics:
  an effective low-energy theory, expansion in the ratio of thermal length 1/T to the typical variation scale L, $\epsilon \equiv \frac{1}{LT}$
- Each term in this derivative expansion is multiplied by an appropriate transport coefficient

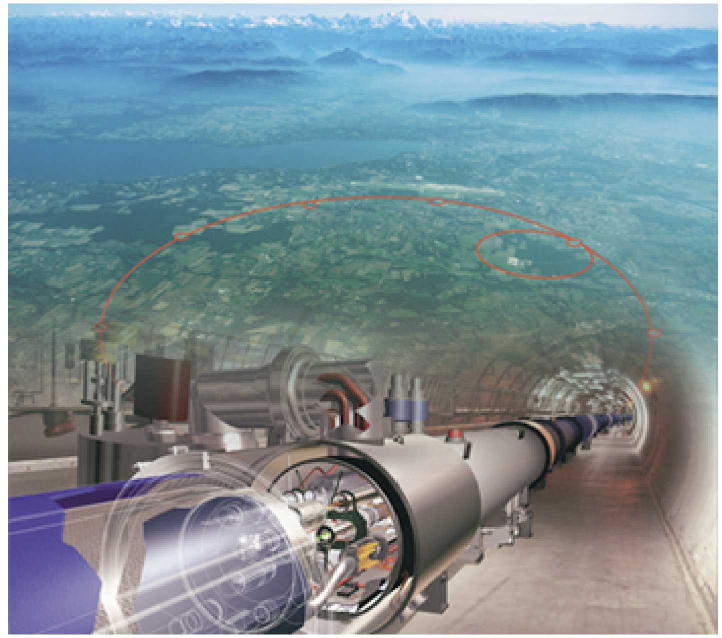
very small shear viscosity - "perfect liquid"; strong coupling





Is there a way to observe topological charge fluctuations in experiment?
yes, in heavy ion collisions!





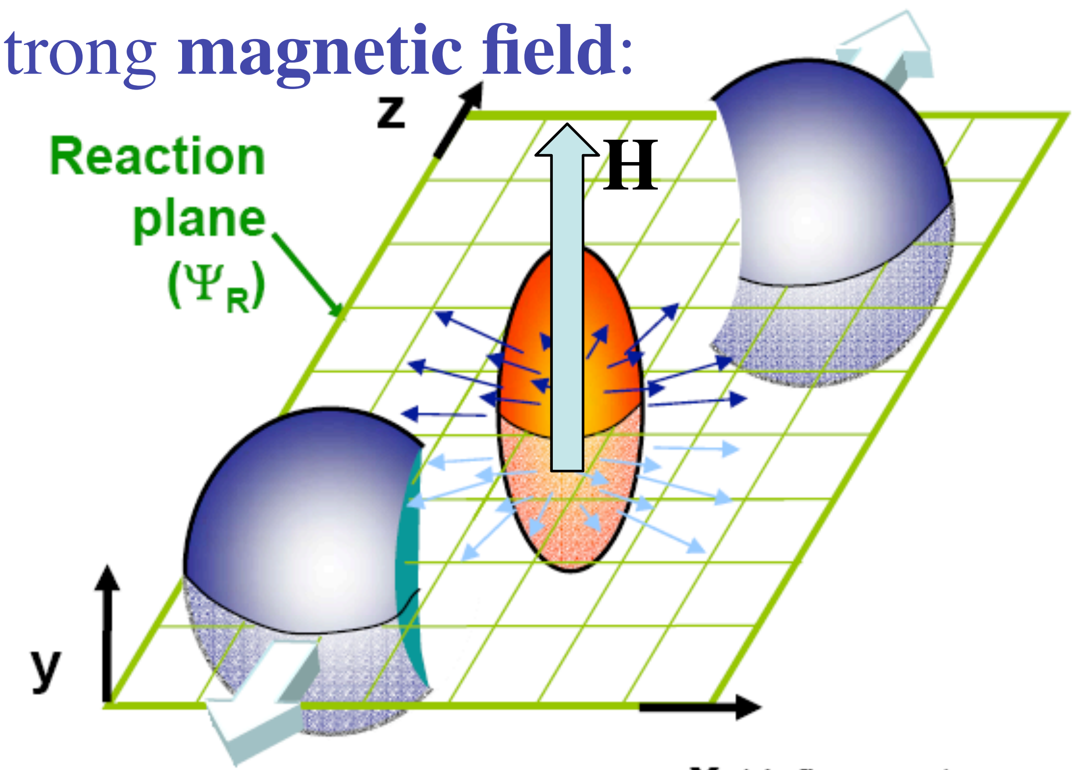
LHC

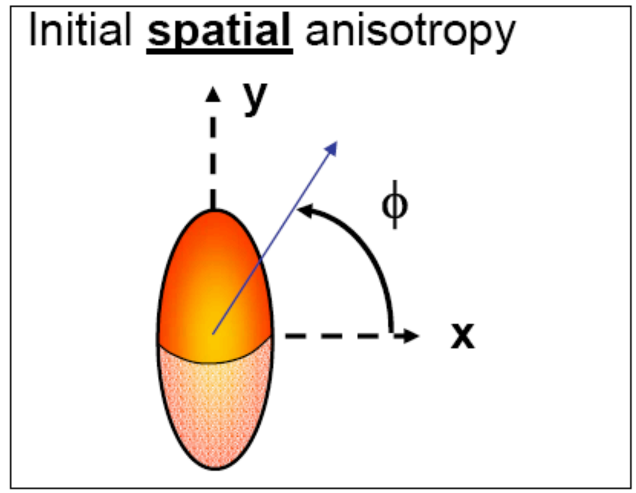
eIC

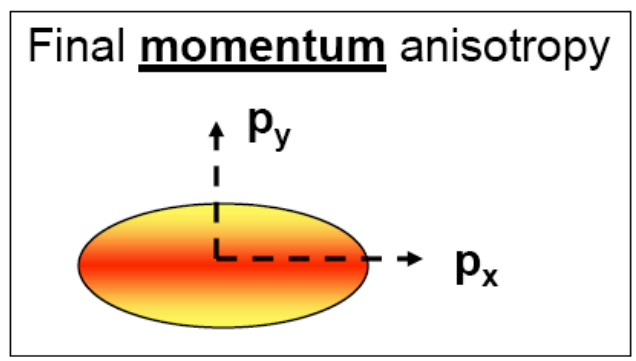
NICA,
JINR

Is there a way to observe topological charge fluctuations in experiment?

Relativistic ions create
a strong **magnetic field**:







Heavy ion collisions as a source of the strongest magnetic fields available in the Laboratory

Also:

V. Skokov,

V. Toneev,

A. Illarionov...

DK, McLerran, Warringa,
Nucl Phys A803(2008)227

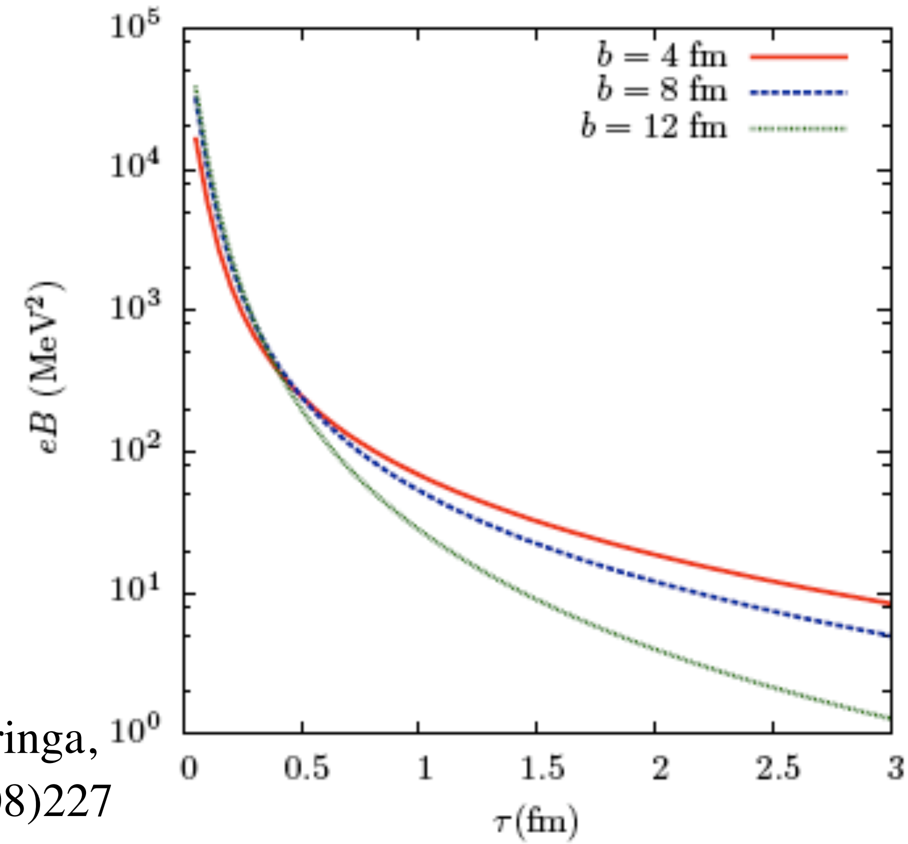


Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

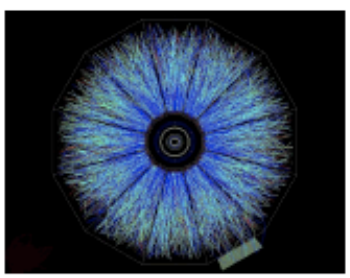
In a conducting plasma, Faraday induction can make the field long-lived:
K.Tuchin, arXiv:1006.3051

NB: magnetic flux is conserved in MHD! - expect the effect at LHC

Comparison of magnetic fields

The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^5 Gauss
The strongest man-made fields ever achieved, if only briefly	10^7 Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$eB(\tau=0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

Chiral Magnetic Effect in a chirally imbalanced plasma

Fukushima, DK, Warringa, PRD'08

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, vector e.m. current is not conserved:

$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left(F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$

Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

The result:

$$\vec{J} = \frac{e^2}{2\pi^2} \; \mu_5 \; \vec{B}$$

Coefficient is fixed by the axial anomaly, no corrections

Axion electrodynamics:
Maxwell-Chern-Simons theory

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\,P_\mu J^\mu_{CS}$$

$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu\theta = (\dot\theta, \vec{P})$$

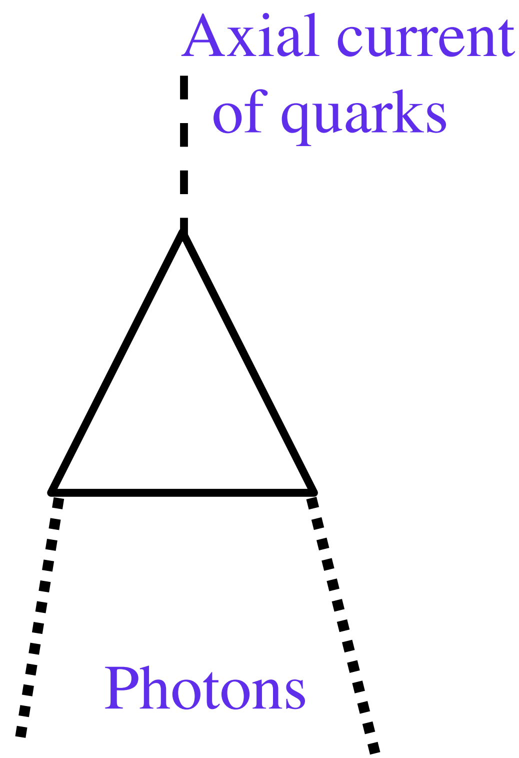
$$\vec\nabla \times \vec B - \frac{\partial \vec E}{\partial t} = \vec J + \underline{c\left(\dot\theta \vec B - \vec P \times \vec E\right)},$$

$$\vec\nabla \cdot \vec E = \rho + \underline{c\vec P \cdot \vec B,}$$

$$\vec\nabla \times \vec E + \frac{\partial \vec B}{\partial t} = 0,$$

$$\vec\nabla \cdot \vec B = 0,$$

EM fields in QCD "aether"



θ - the effective axion field (but no kinetic term)

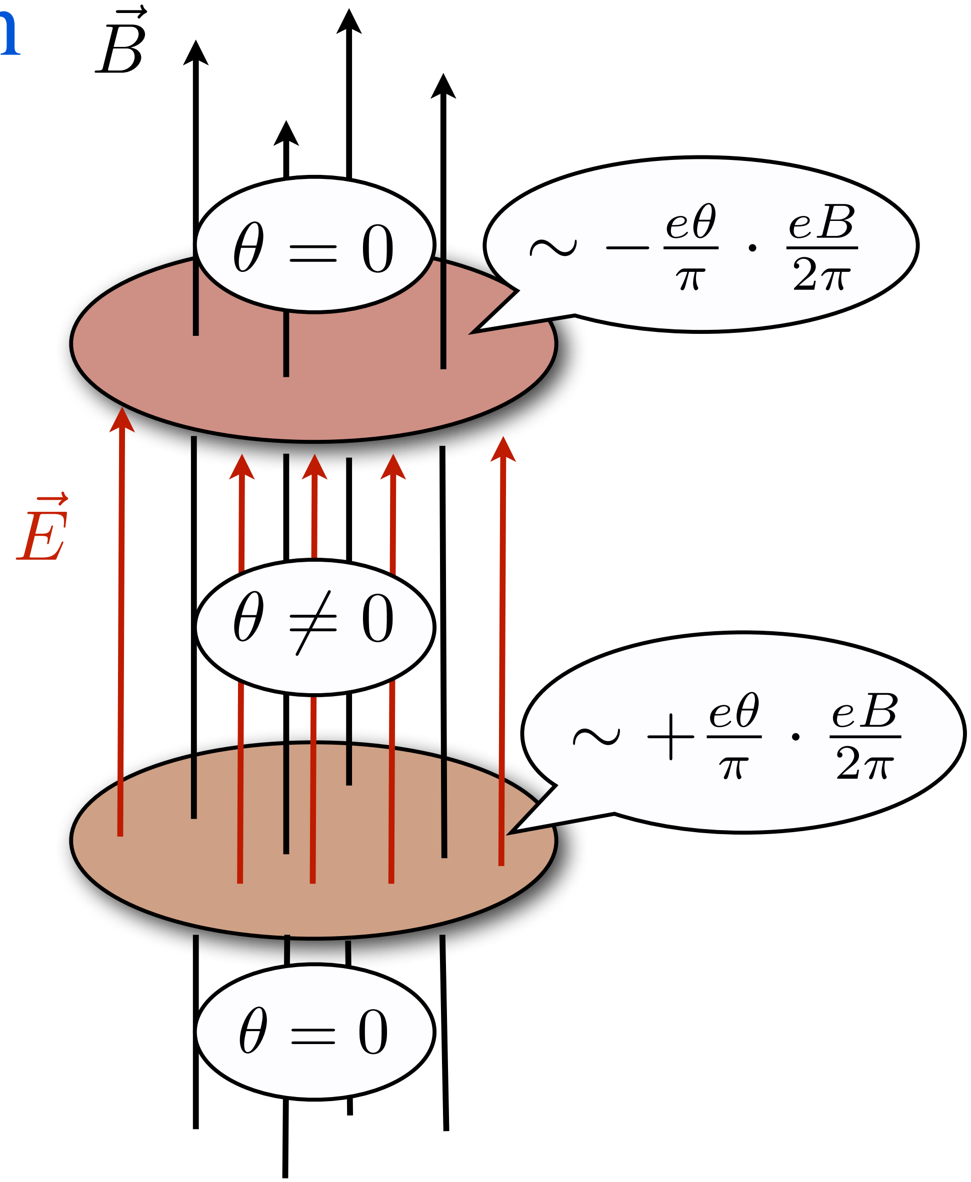
The Chiral Magnetic Effect I:
Charge separation

$$\vec{\nabla} \cdot \vec{E} = \rho + c\vec{P} \cdot \vec{B}$$

$$\vec{P} \equiv \vec{\nabla}\theta$$

$$d_e = \sum_f q_f^2 \left(e\,\frac{\theta}{\pi} \right) \left(\frac{eB \cdot S}{2\pi} \right) L$$

Similar to electric charge
on the axion domain wall

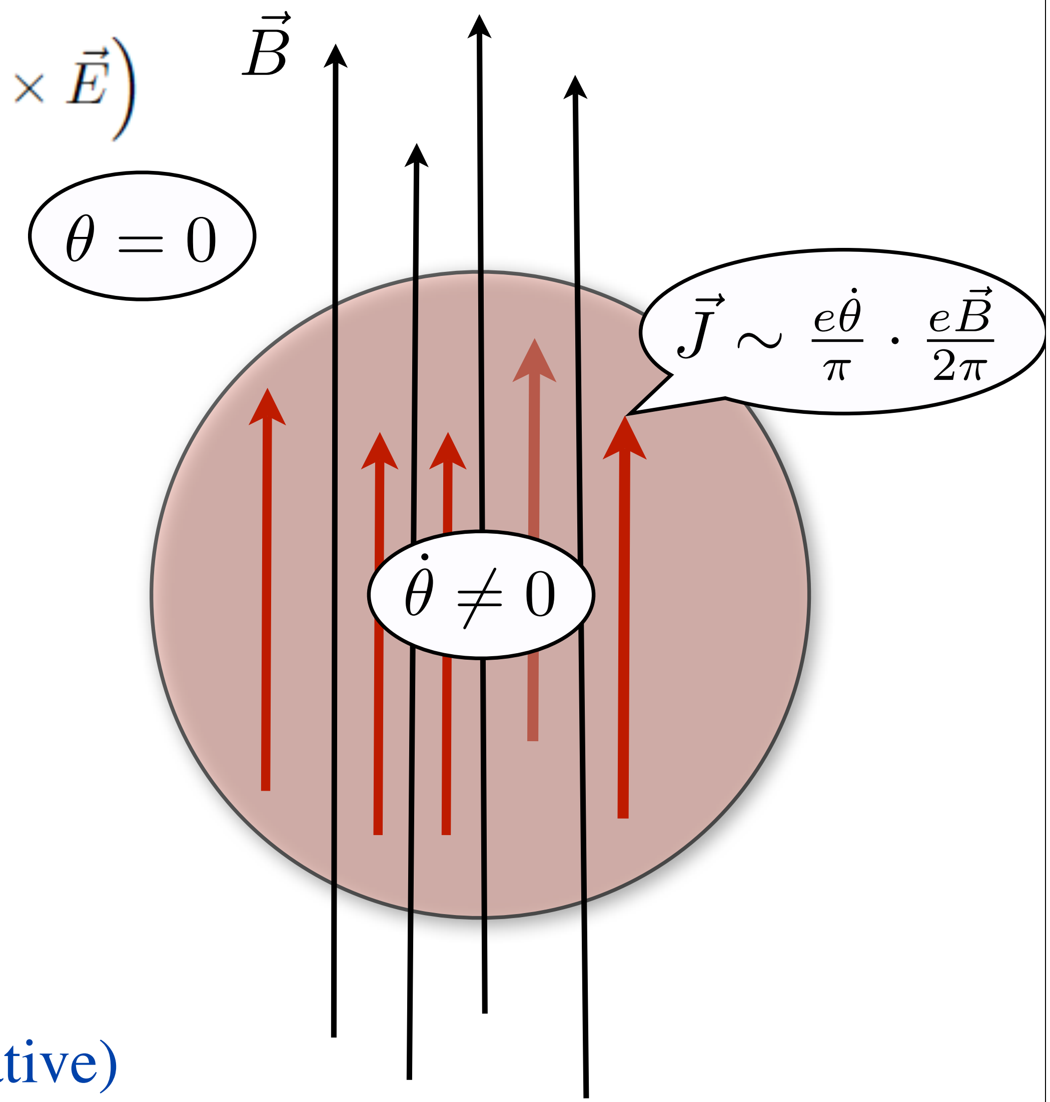


The chiral magnetic effect II: chiral induction

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + c\left(\dot{\theta}\vec{B} - \vec{P} \times \vec{E}\right)$$

$$\vec{J} = -\frac{e^2}{2\pi^2}\dot{\theta}\vec{B}$$

T-even
(reversible,
non-dissipative)



Chiral magnetic conductivity: discrete symmetries

$$\vec{J} = \boxed{\frac{e^2}{2\pi^2}\ \mu_5}\ \vec{B}$$

P-odd
T-odd

P-even
T-odd

P-odd

P-odd effect!

T-even

Non-dissipative current!

(quantum computing etc)

cf Ohmic conductivity:

$$\vec{J} = \sigma \vec{E}$$

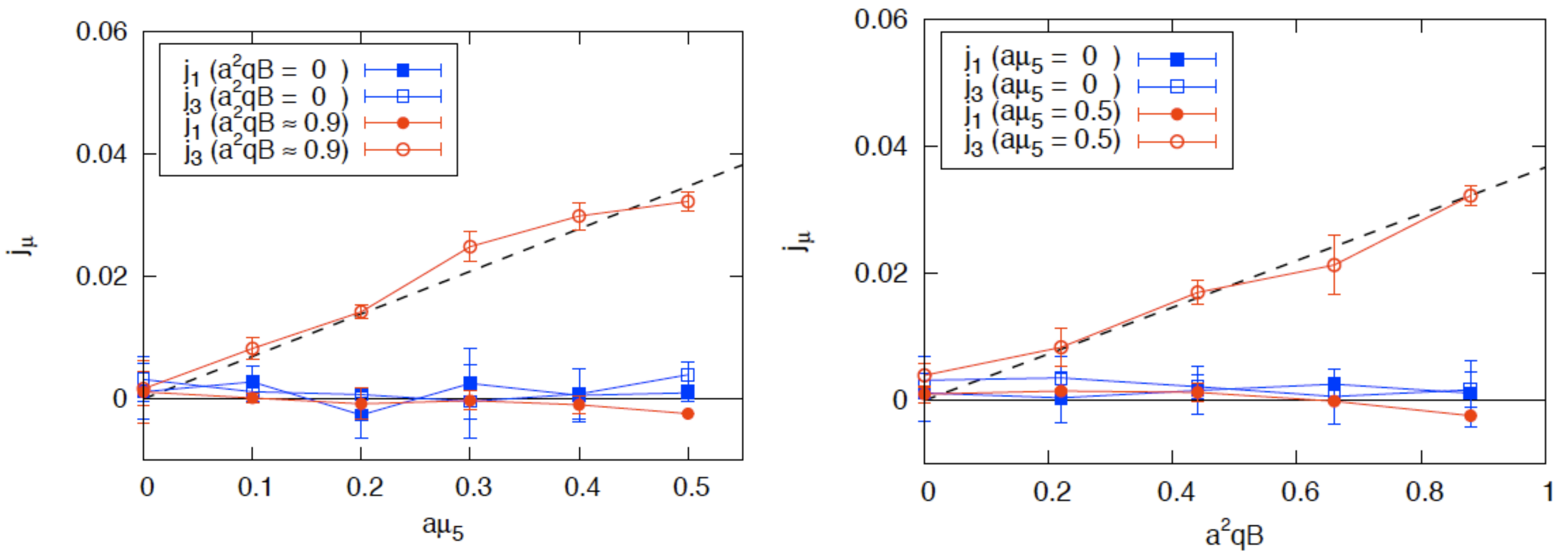
T-odd, dissipative

arXiv:1105.0385

Chiral magnetic effect in lattice QCD with chiral chemical potential

Arata Yamamoto
Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan
(Dated: May 3, 2011)

We perform a first lattice QCD simulation including two-flavor dynamical fermion with chiral chemical potential. Because the chiral chemical potential gives rise to no sign problem, we can exactly analyze a chirally asymmetric QCD matter by the Monte Carlo simulation. By applying an external magnetic field to this system, we obtain a finite induced current along the magnetic field, which corresponds to the chiral magnetic effect. The obtained induced current is proportional to the magnetic field and to the chiral chemical potential, which is consistent with an analytical prediction.



Phase diagram in the (T, μ_5) plane? (no sign problem - ongoing)

Relativistic hydrodynamics and quantum anomalies

- Hydrodynamics: an effective low-energy TOE. States that the response of the fluid to slowly varying perturbations is completely determined by conservation laws (energy, momentum, charge, ...)
- Conservation laws are a consequence of symmetries of the underlying theory
- What happens to hydrodynamics when these symmetries are broken by quantum effects (anomalies of QCD and QED)?

Chiral MagnetoHydroDynamics (CMHD) - relativistic hydrodynamics with triangle anomalies and external electromagnetic fields

First order (in the derivative expansion) formulation:
D. Son and P. Surowka, arXiv:0906.5044

Constraining the new anomalous transport coefficients:
positivity of the entropy production rate, $\partial_\mu s^\mu \geq 0$

$$\nu^\mu = -\sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + \xi\omega^\mu + \xi_B B^\mu,$$

CME (for chirally imbalanced matter)

$$s^\mu = s u^\mu - \frac{\mu}{T}\nu^\mu + D\omega^\mu + D_B B^\mu,$$

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\epsilon + P}\right), \quad \xi_B = C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\epsilon + P}\right).$$

Anomalous terms in hydrodynamics: dictated by 2nd law of thermodynamics!

FLUID MECHANICS

Second Edition

by

L. D. LANDAU and E. M. LIFSHITZ

Institute of Physical Problems, U.S.S.R. Academy of Sciences

Volume 6 of Course of Theoretical Physics
Second English Edition, Revised

Translated from the Russian by
J. B. SYKES and W. H. REID

XV. RELATIVISTIC FLUID DYNAMICS

133. The energy-momentum tensor
134. The equations of relativistic fluid dynamics
...
137. Anomalies in relativistic fluids

should be added to the next editions of hydrodynamics textbooks !

Chiral MagnetoHydroDynamics (CMHD) - relativistic hydrodynamics with triangle anomalies and external electromagnetic fields

First order hydrodynamics has problems with causality and is numerically unstable, so second order formulation is necessary;

Complete second order formulation of CMHD:
DK and H.-U. Yee, 1105.6360

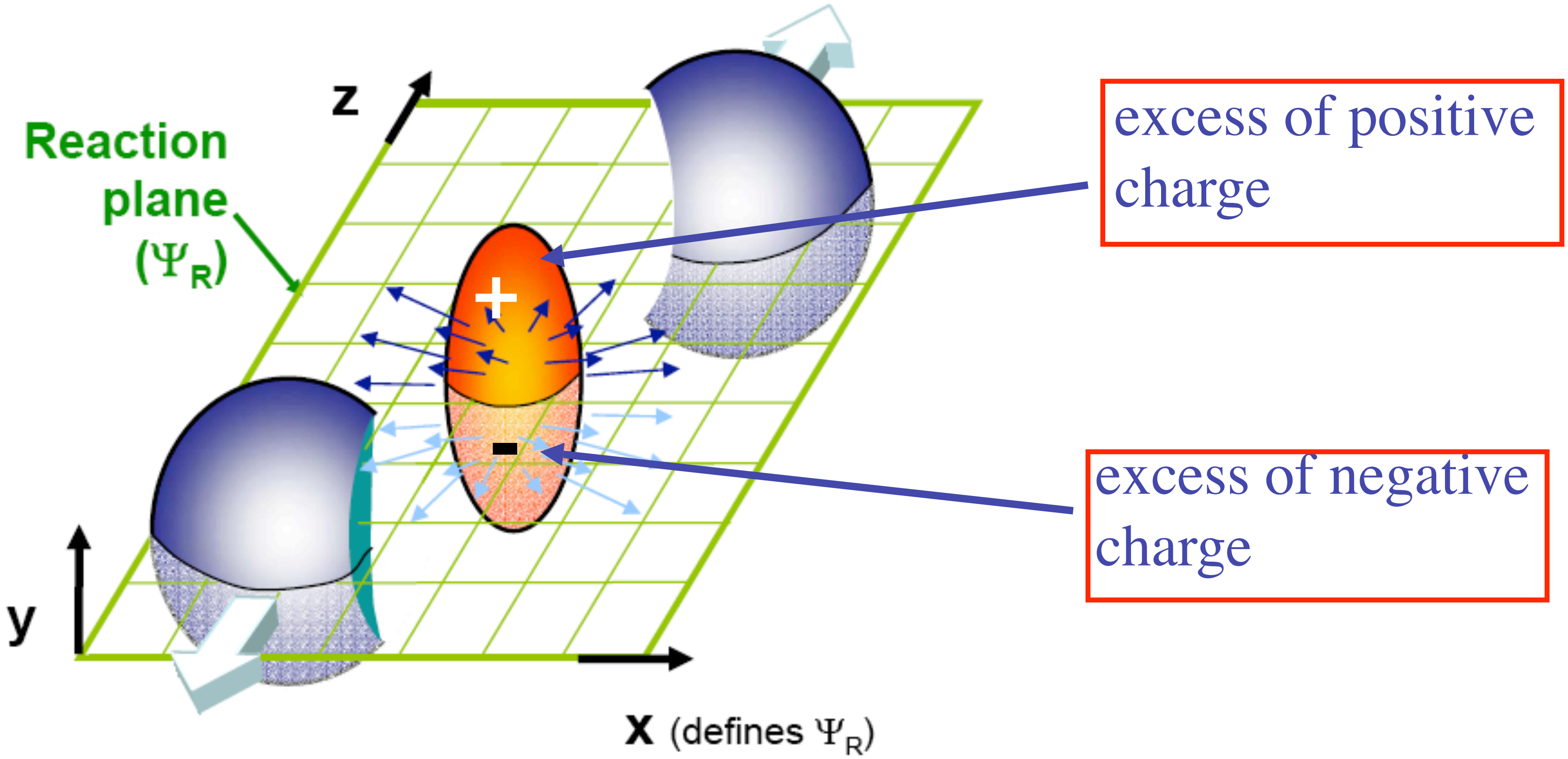
Many new transport coefficients - use conformal/Weyl invariance; still 18 independent transport coefficients related to the anomaly.
15 that are specific to 2nd order; 13 are computed (**T-invariance!**)

new

$$\begin{aligned}
&\sigma^{\mu\nu}\mathcal{D}_\nu\bar{\mu}\ ,\ \omega^{\mu\nu}\mathcal{D}_\nu\bar{\mu}\ ,\ \Delta^{\mu\nu}\mathcal{D}^\alpha\sigma_{\nu\alpha}\ ,\ \Delta^{\mu\nu}\mathcal{D}^\alpha\omega_{\nu\alpha}\ ,\ \sigma^{\mu\nu}\omega_\nu\ ,\\
&\sigma^{\mu\nu}E_\nu\ ,\ \sigma^{\mu\nu}B_\nu\ ,\ \omega^{\mu\nu}E_\nu\ ,\ \omega^{\mu\nu}B_\nu\ ,u^\nu\mathcal{D}_\nu E^\mu\ , \qquad (2.60)\\
&\epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha\mathcal{D}_\beta\bar{\mu}\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu B_\alpha\mathcal{D}_\beta\bar{\mu}\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu E_\alpha B_\beta\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha E_\beta\ ,\ \epsilon^{\mu\nu\alpha\beta}u_\nu\mathcal{D}_\alpha B_\beta\ .
\end{aligned}$$

Many new anomaly-induced phenomena!

How do we look for this in experiment?



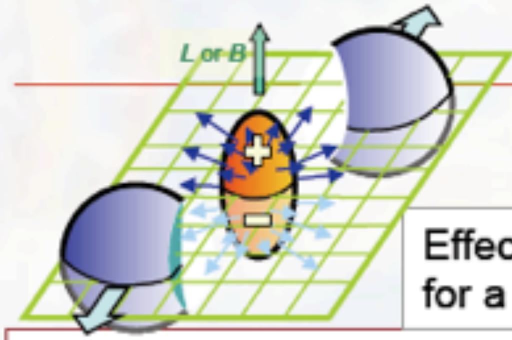
Electric dipole moment of QCD matter!

DK, Phys.Lett.B633(2006)260 [hep-ph/0406125]

Observable

S.A. Voloshin, Phys. Rev. C 70 (2004) 057901

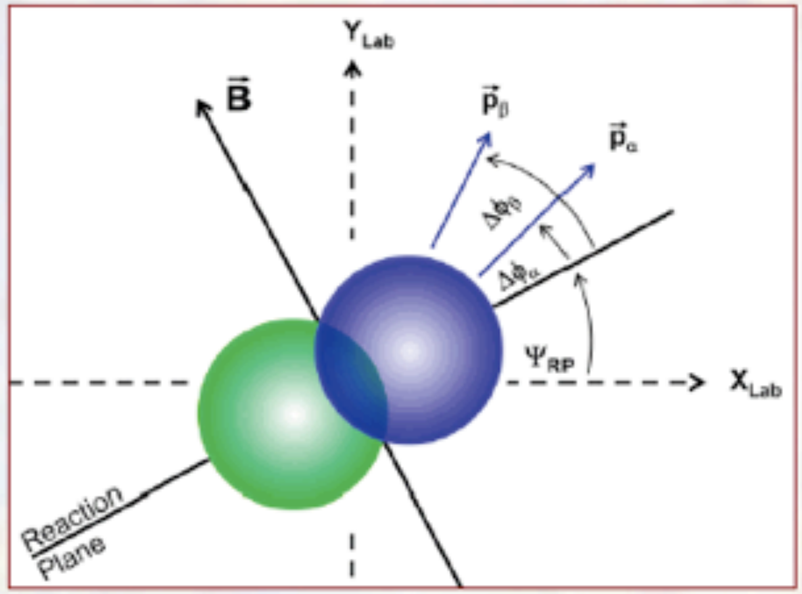
STAR



Effective particle distribution for a certain Q.

$$\frac{dN_\alpha}{d\phi} \propto 1 + 2v_{1,\alpha}\cos(\Delta\phi) + 2\,v_{2,\alpha}\cos(2\Delta\phi) + \dots$$
$$+\ 2a_{1,\alpha}\sin(\Delta\phi) + 2\,a_{2,\alpha}\sin(2\Delta\phi) + \dots,$$

$$\Delta\phi = (\phi - \Psi_{RP})$$



- The effect is too small to observe in a single event
- The sign of Q varies and $\langle a \rangle = 0$ (we consider only the leading, first harmonic) ➔ one has to measure correlations, $\langle a_\alpha a_\beta \rangle$, $\mathcal{P}$-even quantity (!)
- $\langle a_\alpha a_\beta \rangle$ is expected to be ~ 10^{-4}
- $\langle a_\alpha a_\beta \rangle$ can not be measured as $\langle \sin\varphi_\alpha \sin\varphi_\beta \rangle$ due to large contribution from effects not related to the orientation of the reaction plane

➔ study the difference in corr's in- and out-of-plane

Slide from S. Voloshin

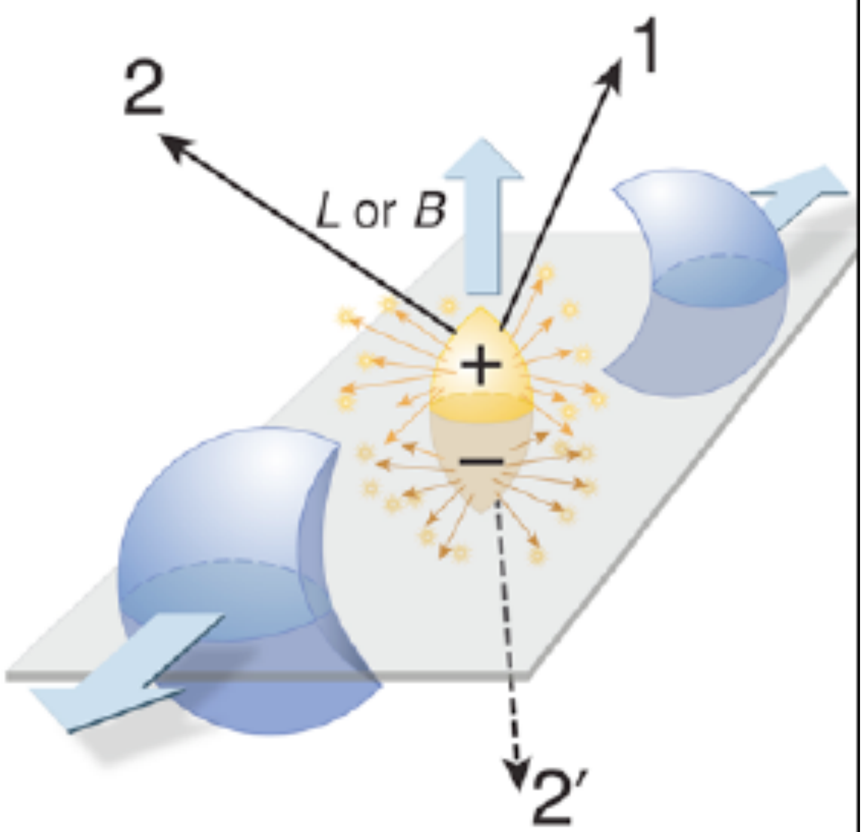
$$\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP})\rangle =$$
$$= \langle\cos\Delta\phi_\alpha\,\cos\Delta\phi_\beta\rangle - \langle\sin\Delta\phi_\alpha\,\sin\Delta\phi_\beta\rangle$$
$$= [\langle v_{1,\alpha}v_{1,\beta}\rangle + B^{in}] - [\langle a_\alpha a_\beta\rangle + B^{out}].$$

$$B^{in} \approx B^{out}, \quad v_1 = 0$$

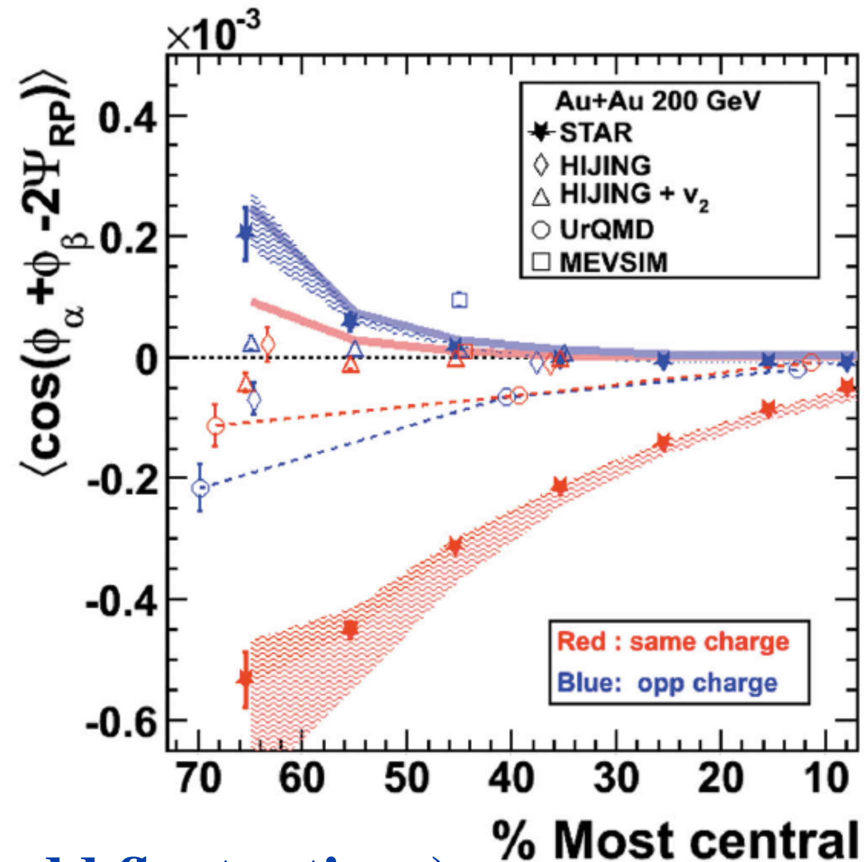
A practical approach: three particle correlations: $\langle\cos(\phi_a + \phi_\beta - 2\phi_c)\rangle = \langle\cos(\phi_a + \phi_\beta - 2\Psi_{RP})\rangle\, v_{2,c}$

PRL 103, 251601 (2009) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2009

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation



(STAR Collaboration)



NB: P-even quantity (strength of P-odd fluctuations)

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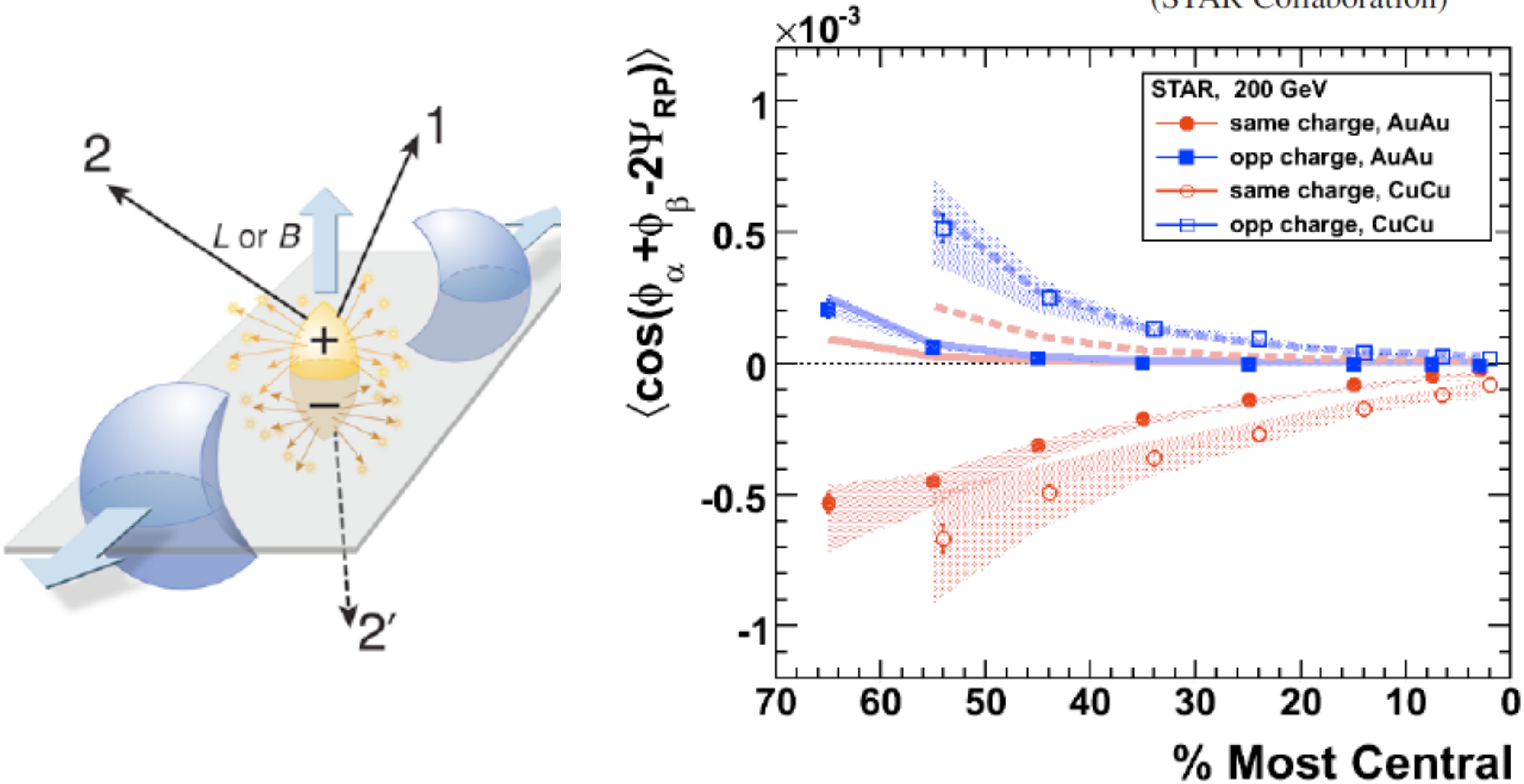
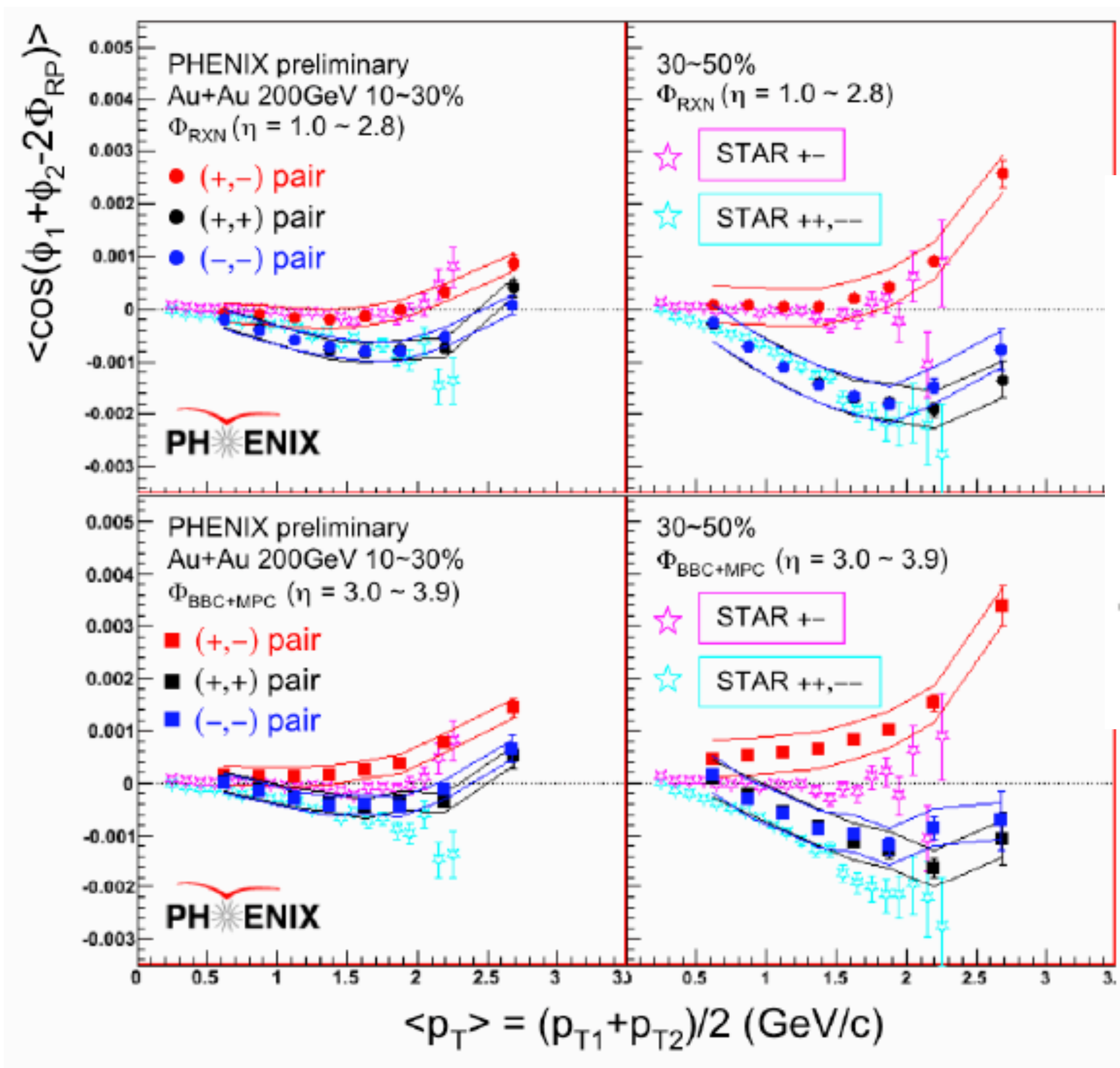
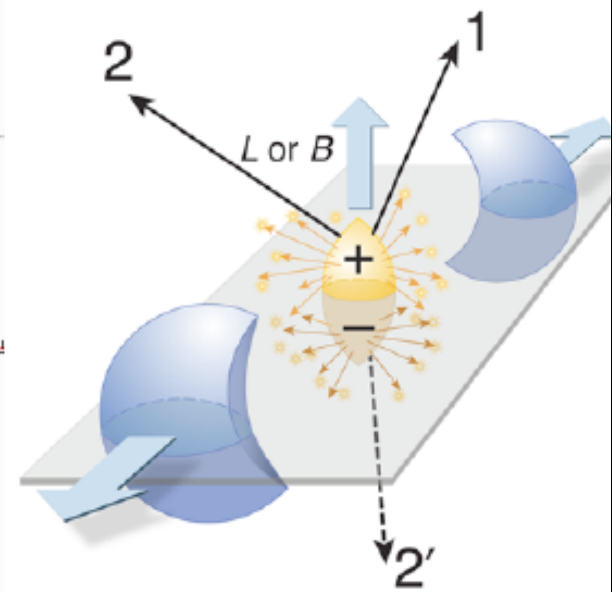


NB: P-even quantity (strength of P-odd fluctuations)



S.Esumi et al
[PHENIX Coll]
April 2010



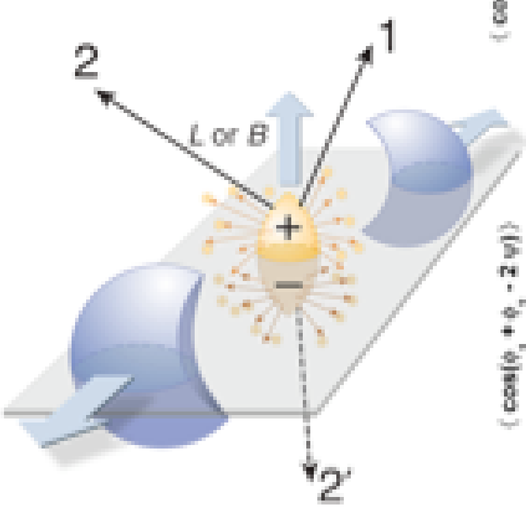
Relatively good agreement between PHENIX & STAR

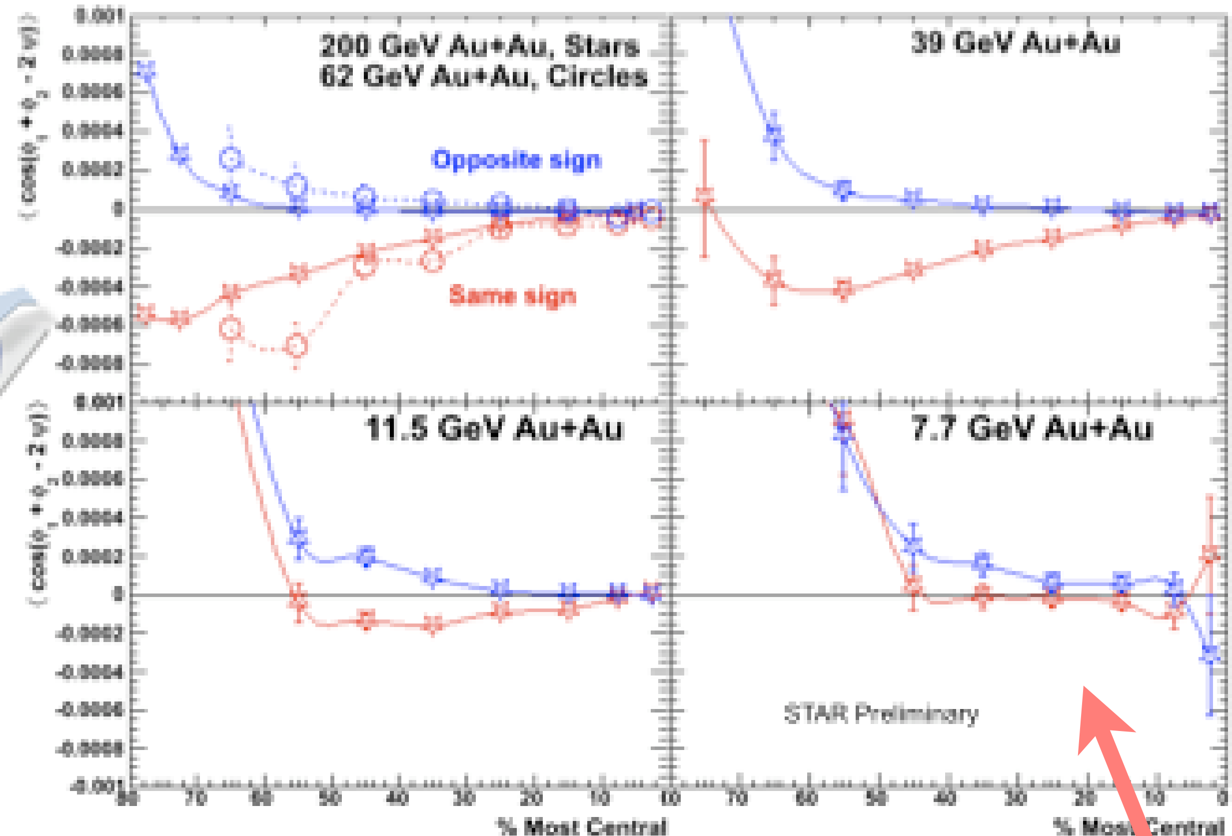
B. Mohanty [STAR Coll] QM 2011

Dynamical Charge Correlations

Observations:

Measurement of charge correlations with respect to event plane

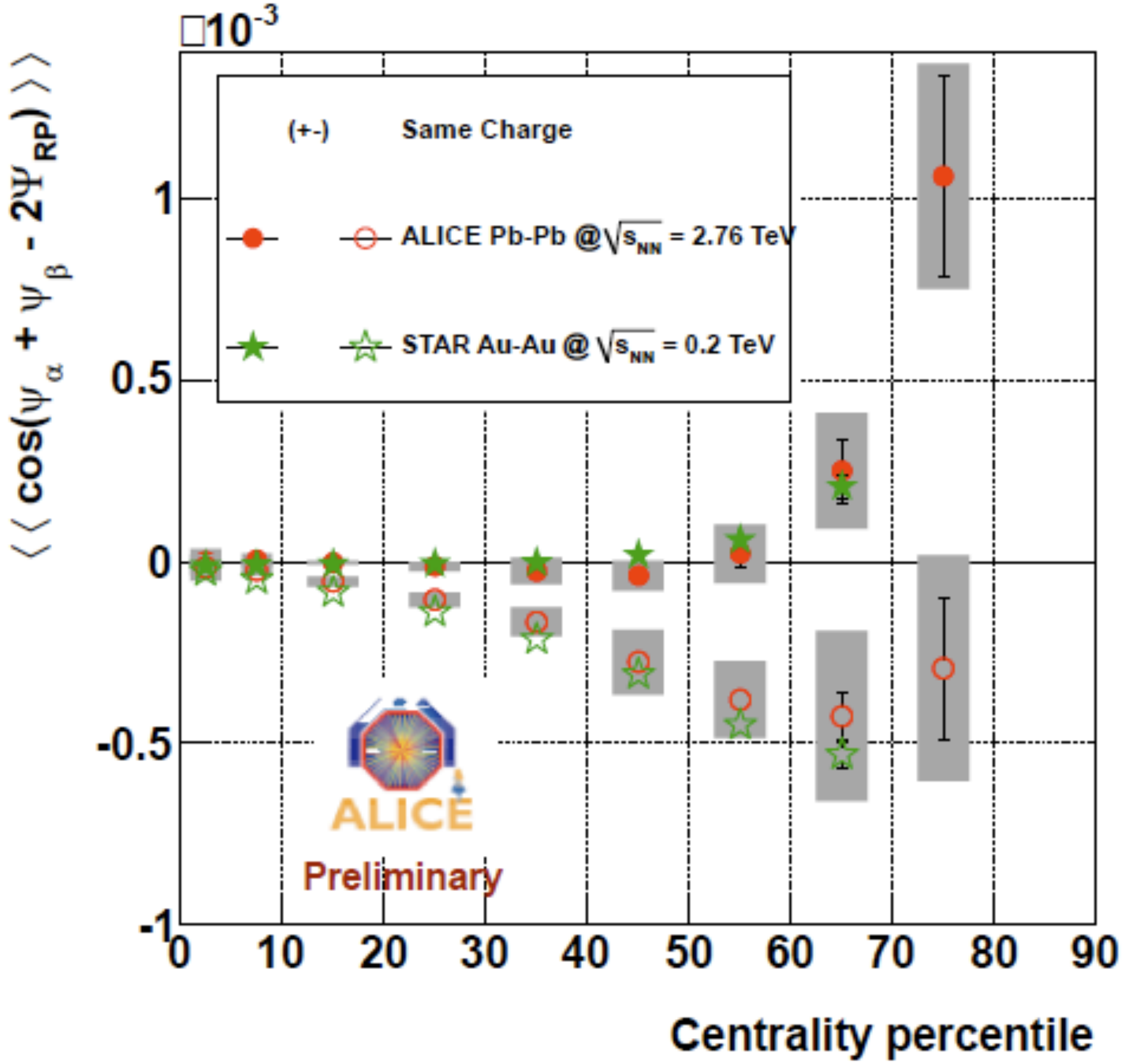




Difference between same sign and opposite sign charge correlations decreases as beam energy decreases.
Same sign charge correlations become positive at 7.7 GeV.

Signal disappears (below T_c)

CME studies at the **LHC**

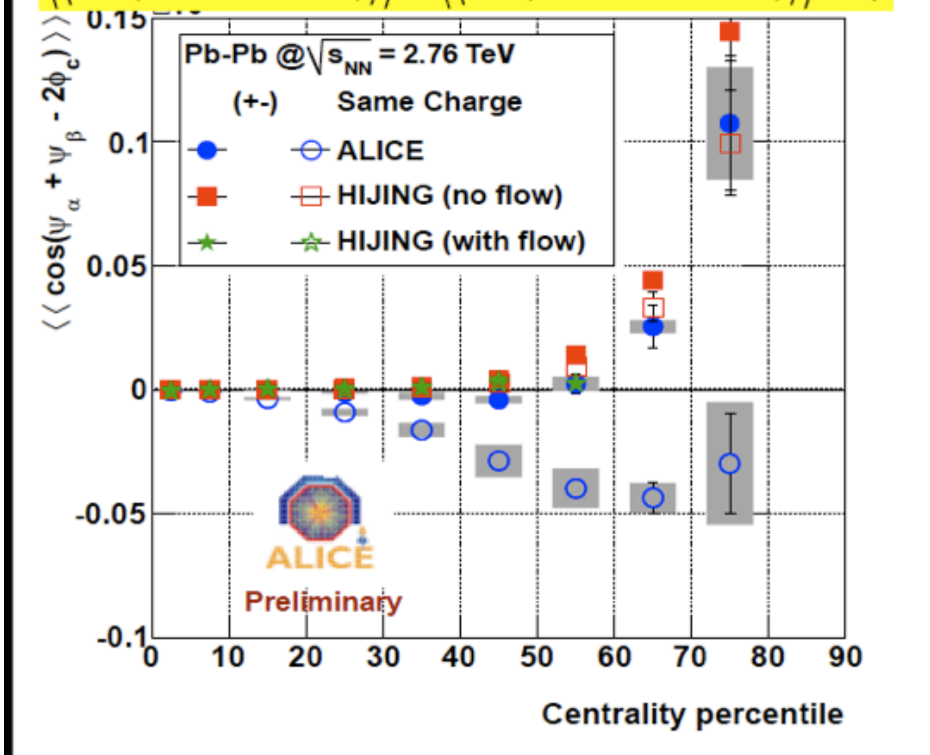


P. Cristakoglou, J. Schukraft [ALICE Coll] Talks at QM 2011

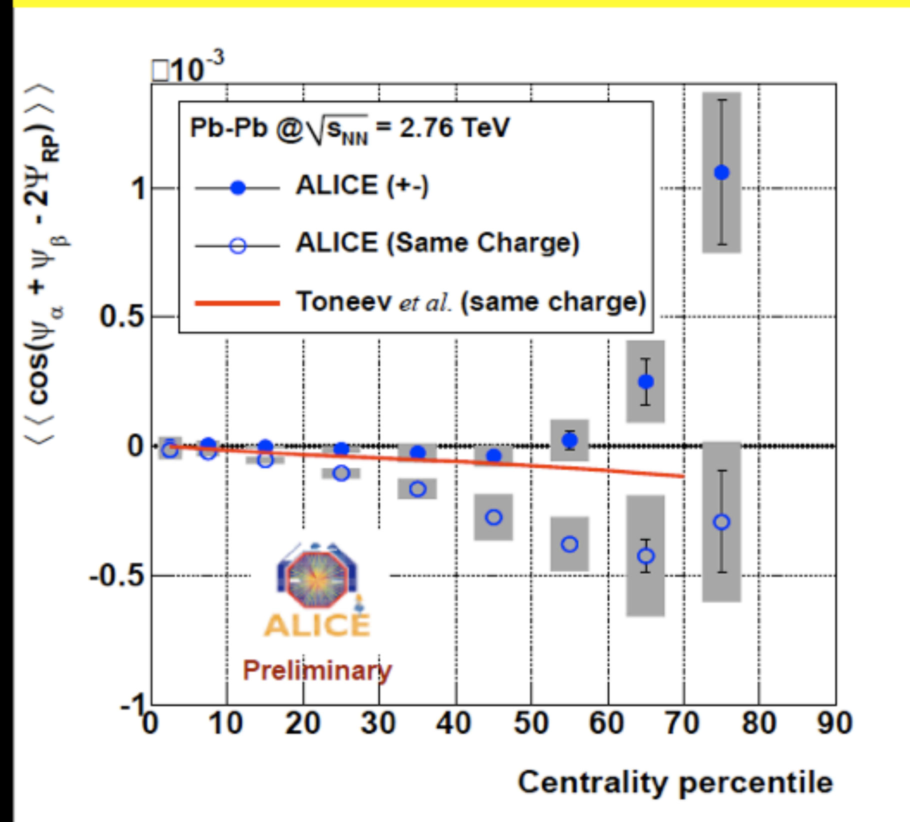
Not reproduced by conventional models

S. A. Voloshin, Phys. Rev. C **70**, 057901 (2004).

$$\langle\langle \cos(\psi_a + \psi_b - 2\phi_c) \rangle\rangle = \langle\langle \cos(\psi_a + \psi_b - 2\Psi_{RP}) \rangle\rangle \square v_{2,c}$$



V.D. Toneev and V. Voronyuk, arXiv:1012.1508v1 [nucl-th]



P. Cristakoglou [ALICE Coll] Talk at QM 2011

A new test: baryon asymmetry

DK, D.T.Son
arXiv:1010.0038; PRL

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2}[\mathrm{tr}(VAQ)\vec{B} + \mathrm{tr}(VAB)2\mu\vec{\omega}]$$

CME

Vorticity-induced
"Chiral Vortical Effect"

$$J_E^{CME} \sim \frac{2}{3} \ \ (N_f = 3) \ \ \text{or} \ \ \frac{5}{9} \ \ (N_f = 2)$$

$$J_B^{CME} = 0 \ \ (N_f = 3) \ \ \text{or} \ \ \sim \frac{1}{9} \ \ (N_f = 2).$$

$$J_E^{CVE} = 0 \ \ (N_f = 3) \ \ \text{or} \ \ \sim \frac{1}{3} \ \ (N_f = 2);$$

$$J_B^{CVE} \sim 1 \ \ (N_f = 3) \ \ \text{or} \ \ \sim \frac{2}{3} \ \ (N_f = 2).$$

CME:
(almost) only
electric charge

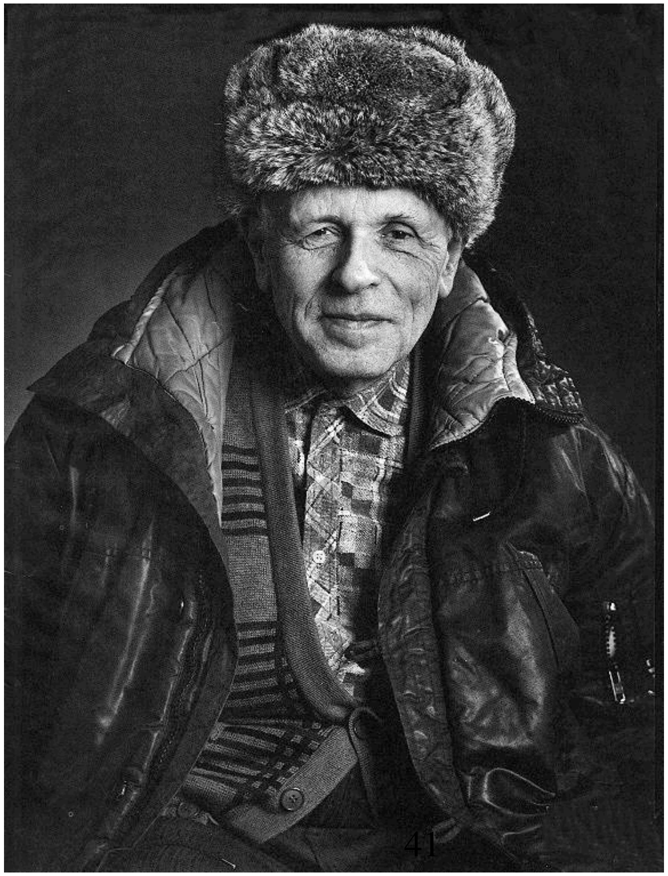
CVE:
(almost) only
baryon charge

There has to be a positive correlation between
electric charge and baryon number! mixed correlators - e.g. $\Lambda \ \ \pi^+$

Cosmic connections: Chirality generation in QGP vs. Baryogenesis in the Early Universe

1. B violation
2. CP violation
3. Non-equilibrium dynamics

A.D. Sakharov,
JETP Lett. 5 (1967) 24



Baryon number ⟷ Chirality
EW sphalerons ⟷ QCD sphalerons
Big Bang ⟷ "Little bang"

If (when) axions are discovered:

- Relativistic plasmas in the Universe have to be described by CMHD coupled to the (space-time dependent) axion field

- Novel mechanisms for the generation of primordial magnetic field, separation of matter from anti-matter, polarization of CMB, acceleration of UHE particles, ...

Summary

Interplay of topology, anomaly and magnetic field leads to the Chiral Magnetic Effect: confirmed by lattice QCD x QED, signature of chiral symmetry restoration

CME and related anomaly-induced phenomena are an integral part of relativistic hydrodynamics (Chiral MagnetoHydroDynamics)

Experimental evidence at RHIC at LHC; more studies underway